

Iran University of Science & Technology

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Digital Logic Design

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Switching Functions

• Switching algebra

- Boolean algebra with the set of elements $K = \{0, 1\}$
- If there are *n* variables, we can define 2^{2^n} switching functions.

AB	f_0	f_l	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}
00	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
01	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
10	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
11	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

•A switching expression as follows:

• $f_0(A,B) = 0$, $f_6(A,B) = AB' + A'B$, $f_{11}(A,B) = AB + A'B + A'B' = A' + B$, ...



Some Definition: 1

•Literal

- A variable, complemented or uncomplemented
- $\circ A, \overline{A}, B, \overline{B}$

Product term

- A literal or literals ANDed together
- $(A \cdot B \cdot \overline{C})$, $(\overline{A} \cdot C)$, $(B \cdot \overline{C})$

•Sum term

- A literal or literals ORed together.
- $(A + B + \overline{C}), (\overline{A} + C), (B + \overline{C})$

• Minterm

- A product that includes all the variables
- $(A \cdot B \cdot \overline{C})$, $(\overline{A} \cdot \overline{B} \cdot C)$, $(\overline{A} \cdot B \cdot \overline{C})$

Maxterm

A sum that includes all the variables

•
$$(A + B + \overline{C})$$
, $(\overline{A} + \overline{B} + C)$, $(\overline{A} + B + \overline{C})$



Algebraic Forms

•SOP

- Sum of Products
- ORing product terms
- f(A, B, C) = ABC + A'C + B'C

•POS

- Product of Sums
- ANDing sum terms
- f(A, B, C) = (A' + B' + C')(A + C')(B + C')

Canonical SOP

- Represented as a sum of minterms only.
- Example: $f_1(A,B,C) = A'BC' + ABC' + A'BC + ABC$

Canonical POS

- Represented as a product of maxterms only.
- Example: f₂(A,B,C) = (A+B+C)(A+B+C')(A'+B+C)(A'+B+C')

0



Outline

• Logic Gates



Transistors





Transistor

• Computers are built from very large numbers of very simple structures

- Intel's Pentium IV microprocessor, first offered for sale in 2000, was made up of more than 42 million MOS transistors
- Intel's Core i7 Broadwell-E, offered for sale in 2016, is made up of more than 3.2 billion MOS transistors





Transistor Structure

- Structure of a typical transistor
- A transistor conducts when the channel is filled with carriers
 - Negative carriers (free electrons)
 - Positive carriers (holes electrons)
- Why is this useful?
 - We can combine many of these to realize simple logic gates





BJT Transistor

• Bipolar Junction Transistor (BJT)





MOS Transistor

•Field Effect Transistors (FET)

- PMOS: if Gain =0 \rightarrow electron flow from drain to source
- NMOS: if Gain =1 \rightarrow electron flow from drain to source







How Does a Transistor Work?

- In order for the lamp to glow, electrons must flow
- In order for electrons to flow, there must be a closed circuit from the power supply to the lamp and back to the power supply
- The lamp can be turned on and off by simply manipulating the wall switch to make or break the closed circuit





Switches

- A switch has two states
 - Closed/ On
 - Open/OFF











Switches: Sample 1





Switches: Sample1 (cont'd)

- Find L(A,B,C,D)
 - A (B \overline{C} + D)
 - A B \overline{C} + A D









Switching and Transistors

 Instead of the wall switch, we could use an n-type or a p-type MOS transistor to make or break the closed circuit



If the gate of an n-type transistor is supplied with a high voltage, the connection from source to drain acts like a piece of wire

Depending on the technology, 0.3V to 3V

If the gate of the -type transistor is supplied with OV, the connection between the source and drain is broken



How Does a Transistor Work?

• The n-type transistor in a circuit with a battery and a bulb





How Does a Transistor Work?

 The p-type transistor works in exactly the opposite fashion from the n-type transistor



Logic gates

One Level Higher in Abstraction



- How do we build logic out of MOS transistors?
- We construct basic logic structures out of individual MOS transistors
- These logical units are named logic gates
 - They implement simple **Boolean** functions

	-
Problem	
Algorithm	
Program/Language	
Runtime System (VM, OS, MM)	
ISA (Architecture)	
Microarchitecture	
Logic	
Devices	
Electrons	



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Making Logic Blocks Using CMOS Technology

 Modern computers use both n-type and p-type transistors, i.e. Complementary MOS (CMOS) technology

nMOS + pMOS = CMOS

•The simplest logic structure that exists in a modern computer



Functionality of Our CMOS Circuit

What happens when the input is connected to 0V?



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Functionality of Our CMOS Circuit



What happens when the input is connected to 3V?





CMOS NOT Gate

- This is actually the CMOS NOT Gate
- Why do we call it NOT?
 - If A = 0V then Y = 3V
 - If A = 3V then Y = 0V
- Digital circuit: one possible interpretation
 - Interpret OV as logical (binary) O value
 - Interpret 3V as logical (binary) 1 value







CMOS NOT Gate



- - What would be the logical output of the circuit for each possible input



Logic Gates: NOT

- (a) NOT logic function.
- (b) Electronic NOT gate.
- (c) Standard symbol.
- (d) IEEE block symbol.





Another CMOS Gate: What Is This?

• Let's build more complex gates!





CMOS NAND Gate







A	B	P1	P2	N1	N2	Y
0	0					
0	1					
1	0					
1	1					





• P1 and P2 are in parallel; only one must be ON to pull the output up to 3V



Let's build more complex gates! $Y = \overline{A \cdot B} = \overline{AB}$ 3V **P1 P2 N2 N1** Y B Α **P2** Out (Y) OFF 0 0 ON ON OFF 1 **N1** In (A) OFF 1 0 1 ON OFF ON **N2** In (B) 1 0 1 1 ∇

CMOS NAND Gate

P1 and P2 are in parallel; only one must be ON to pull the output up to 3V



Let's build more complex gates! $Y = \overline{A \cdot B} = \overline{AB}$ 3V **P2 N2 P1** B **N1** Y Α **P2** Out (Y) ON ON OFF OFF 1 0 0 **N1** In (A) OFF 1 1 ON OFF ON 0 **N2** In (B) 1 1 0 OFF ON ON OFF 1 1 ∇

CMOS NAND Gate

- P1 and P2 are in parallel; only one must be ON to pull the output up to 3V
- N1 and N2 are connected in series; both must be ON to pull the output to OV



Let's build more complex gates! $Y = \overline{A \cdot B} = \overline{AB}$ 3V **P2 N2 P1** Y B **N1** Α **P2** Out (Y) ON ON OFF OFF 1 0 0 **N1** In (A) 1 1 ON OFF OFF ON 0 **N2** In (B) 1 1 0 OFF ON ON OFF 0 1 OFF OFF ON ON ∇

CMOS NAND Gate

- P1 and P2 are in parallel; only one must be ON to pull the output up to 3V
- N1 and N2 are connected in series; both must be ON to pull the output to OV



CMOS NAND Gate

• Let's build more complex gates!





Logic Gates: NAND

- •(a) NAND logic function
- •(b) Electronic NAND gate
- •(c) Standard symbol
- •(d) IEEE block symbol

a b	$fNAND (a, b) = \overline{ab}$	A	B	Y
0 0	1	L	L	Н
0 1	1	L	Η	Η
1 0	1	Η	L	Η
1 1	0	Н	Η	L
	(a)		(b)	





Logic Gates: NAND: Properties $f_{NAND}(a,a) = a \cdot a = \overline{a} = f_{NOT}(a)$ \overline{a} $f_{NAND}(\overline{a}, \overline{b}) = \overline{a} \cdot \overline{b} = a + b = f_{OR}(a, b)$ a·b a·b b $\overline{f}_{NAND}(a,b) = a \cdot b = a \cdot b = f_{AND}(a,b)$ a+b $a \cdot b = a + b$

• NAND gate may be used to implement all three elementary operators.



NAND: Universal Gate

•Universal gate

- A gate type that can implement any Boolean function.
- NAND is a universal gate



AND gate






Logic Gates: NAND (cont'd)

- NAND operation is NOT associative
- (X NAND Y) NAND $Z \neq X$ NAND (Y NAND Z)







CMOS AND Gate

- How can we make a AND gate?
- Let's check NAND gate



$$Y = A \cdot B = AB$$







CMOS AND Gate

How can we make a AND gate?





Logic Gates: AND

- (a) AND logic function.
- (b) Electronic AND gate.
- (c) Standard symbol.
- (d) IEEE block symbol.





Logic Gates: OR

- (a) OR logic function.
- (b) Electronic OR gate.
- (c) Standard symbol.
- (d) IEEE block symbol.





Logic Gates: NOR

- •(a) NAND logic function
- •(b) Electronic NAND gate
- •(c) Standard symbol
- •(d) IEEE block symbol

a	b	$f_{NOR}(a, b) = \overline{a + b}$	A B Y
0	0	1	L L H
0	1	0	L H L
1	0	0	H L L
1	1	0	H H L
		(a)	(b)





Logic Gates: NOR: Properties

NOR gate may be used to implement all three elementary operators
NOR is universal gate

$$f_{NOR}(a,a) = \overline{a+a} = \overline{a} = f_{NOT}(a)$$
$$\overline{f_{NOR}(a,b)} = \overline{\overline{a+b}} = a+b = f_{OR}(a,b)$$
$$f_{NOR}(\overline{a},\overline{b}) = \overline{\overline{a+b}} = a \cdot b = f_{AND}(a,b)$$



NOT gate



OR gate



AND gate



Logic Gates: NOR (cont'd)

- NOR operation is NOT associative
- (X NOR Y) NOR $Z \neq X$ NOr (Y NOR Z)





















Logic Gates: Sample 3

• \geq 1 in IEEE symbol

ab	sum(a, b)	$\operatorname{sum}(a, b) \ge 1$	$f_{OR}(a, b) = a + b$
00	0	False	0
01	1	True	1
10	1	True	1
11	2	True	1



Logic Gates: Sample 1

• Implement a device which takes two inputs and indicates whether their sum is greater than 1 or not?

ab	sum(a, b)	$\operatorname{sum}(a, b) \ge 1$
00	0	False
01	1	True
10	1	True
11	2	True



Logic Gates: Sample 1

• Implement a device which takes two inputs and indicates whether their sum is greater than 1 or not?

ab	sum(a, b)	$sum(a, b) \ge 1$	$f_{OR}(a, b) = a + b$
00	0	False	0
01	1	True	1
10	1	True	1
11	2	True	1



Logic Gates: XOR

- Exclusive-OR (XOR)
- •(a) XOR logic function
- •(b) Electronic XOR gate
- •(c) Standard symbol
- •(d) IEEE block symbol

a h	$f(a,b) = a \oplus b$	ΔR	V
u U	$f_{XOR}(a, b) = a \oplus b$	MD	1
0 0	0	LL	L
01	1	LH	Η
10	1	ΗL	Η
11	0	ΗH	L



 $A \oplus B = \overline{A} \cdot B + \overline{B} \cdot A$



Logic Gates: XOR (cont'd)

POS of XOR
 a ⊕ b

 $= \overline{a}b + a\overline{b}$ $= \overline{a}a + \overline{a}b + a\overline{b} + b\overline{b}$ $= \overline{a}(a+b) + \overline{b}(a+b)$ $= (\overline{a} + \overline{b})(a+b)$

- Some useful relationships
 - $\circ a \oplus a = 0$
 - $\circ a \oplus \overline{a} = 1$
 - $\circ a \oplus 0 = a$
 - $a \oplus 1 = \overline{a}$
 - $\circ \overline{a} \oplus \overline{b} = a \oplus b$
 - $\circ a \oplus b = b \oplus a$
 - $a \oplus (b \oplus c) = (a \oplus b) \oplus c$



Logic Gates: Sample 2

- Implement a device which takes two inputs and indicates whether their sum is greater than 1 or not?
- Inputs and outputs are 1 bit.

ab	sum(a, b)	sum(a, b) = 1?
00	0	False
01	1	True
10	1	True
11	2	False



Logic Gates: Sample 2

- Mathematical sum of inputs is one
 - Output of XOR gate is asserted
 - Output of XOR is the *modulo*-2 sum of its inputs.

ab	sum(a, b)	sum(a, b) = 1?	$f(a, b) = a \oplus b$
00	0	False	0
01	1	True	1
10	1	True	1
11	2	False	0



Logic Gates: XNOR

- Exclusive-NOR (XNOR)
- •(a) XOR logic function
- •(b) Electronic XOR gate
- •(c) Standard symbol
- •(d) IEEE block symbol



$$\overline{\mathbf{A} \oplus \mathbf{B}} = \mathbf{A} \odot \mathbf{B} = \overline{\overline{\mathbf{A}} \cdot \mathbf{B} + \overline{\mathbf{B}} \cdot \mathbf{A}}$$



Logic Gates: XNOR: POS

• POS of XOR

 $a \oplus b$

- $= \overline{a}a + \overline{a}b + a\overline{b} + b\overline{b}$ $= \overline{a}(a+b) + \overline{b}(a+b)$ $= (\overline{a} + \overline{b})(a+b)$
- Some useful relationships

 $=\overline{a}b+a\overline{b}$

- *a* ⊕ *a* = 0
- $\circ a \oplus \overline{a} = 1$
- $a \oplus 0 = a$
- $a \oplus 1 = \overline{a}$
- $\overline{a} \oplus \overline{b} = a \oplus b$
- $a \oplus b = b \oplus a$
- $a \oplus (b \oplus c) = (a \oplus b) \oplus c$



Logic Gates: XNOR: SOP

SOP and POS of XNOR

$$a \odot b = \overline{a \oplus b}$$

$$= \overline{a \oplus b}$$

$$= \overline{ab + a\overline{b}}$$

$$= \overline{ab} \cdot \overline{ab}$$

$$= (a + \overline{b})(\overline{a} + b)$$

$$= a\overline{a} + ab + \overline{a}\overline{b} + \overline{b}b$$

$$= ab + \overline{a}\overline{b}$$

Logic Gates: XNOR: Is it Correct?



 $a \oplus \overline{b} = a \odot b$

Logic Gates: XNOR: Is it Correct?



 $a \oplus \overline{b} = a \odot b$

$$\mathbf{a} \oplus \overline{b} = \overline{a} \cdot \overline{b} + a \cdot b$$
$$\mathbf{a} \odot \mathbf{b} = \mathbf{a} \cdot b + \overline{a} \cdot \overline{\mathbf{b}}$$



Logic Gates: Buffer

•F(x)= x

•Boolean function is a connection.

•Usage

- Amplify an input signal
- Permits more gates to be attached to output







Logic Gates: Hi-Impedance

- Logic gates have 0 or 1 as output; Two-state logic
 - Their output cannot connect together
- Three-state logic:
 - Adds a third value; Hi-Impedance output, Hi-Z
 - Three output values: 1, 0, and Hi-Z
 - Output appears to be disconnected from the input
 - Behaves as an open circuit between gate input & output
 - Hi-impedance gates can connect their outputs together



Logic Gates: 3-State Buffer

 Two inputs IN • Data input (IN) OUT Enable control input (EN) EN • One output output (OUT) EN IN OUT Hi-Z Χ 0 1 0 0 1 1 1 closed open Q = Z Q = A Enable = "0" Enable = "1"



• Active high





Active low





Logic Gates: 3-State Buffer (cont'd)

- Output of 3-state buffers can be wired together
- At most one 3-state buffer can be enabled.
 - Resolved output is equal to the output of the enabled 3-state buffer
- If multiple 3-state buffers are enabled at the same time
 - Conflicting outputs will burn the circuit



Resolution Table			
00	01	02	OUT
0 or 1	Hi-Z	Hi-Z	00
Hi-Z	0 or 1	Hi-Z	01
Hi-Z	Hi-Z	0 or 1	02
Hi-Z	Hi-Z	Hi-Z	Hi-Z
0 or 1	0 or 1	0 or 1	Burn



3-State Buffer & Data Bus

Isolate devices and circuits from the data bus and one another











Logic Gates: Types

- Signals and logic values
 - A signal that is set to logic 1 is said to be asserted, active, or true.
 - An active-high signal is asserted when it is high (positive logic).
 - An active-low signal is asserted when it is low (negative logic).
- Positive logic
- Negative logic

Electric Signal	Logic Value		
	Positive Logic	Negative Logic	
High Voltage (H)	1	0	
Low Voltage (L)	0	1	



Negative Logic: AND

- (a) AND gate truth table (L=1, H=0)
- (b) Alternate AND gate symbol (in negative logic)
- (c) Preferred usage
- (d) Improper usage





Logic Gates: Negative Logic: AND (cont'd)

y = a . b =
$$\overline{\overline{a \cdot b}} = \overline{\overline{a} + \overline{b}} = \overline{f}_{OR}(\overline{a}, \overline{b})$$

 $\overline{y} = \overline{\overline{(\overline{a})} + \overline{(\overline{b})}} = \overline{a + b} = \overline{f}_{OR}(a, b)$





Logic Gates: Negative Logic: OR

- (a) OR gate truth table (L=1, H=0)
- (b) Alternate OR gate symbol (in negative logic)
- (c) Preferred usage
- (d) Improper usage



Logic Gates: Negative Logic: OR (cont'd)



$$y = a + b = \overline{\overline{a + b}} = \overline{\overline{a \cdot \overline{b}}} = \overline{f}_{AND}(\overline{a}, \overline{b})$$
$$\overline{y} = \overline{\overline{(\overline{a})} \cdot \overline{(\overline{b})}} = \overline{a \cdot b} = \overline{f}_{AND}(a, b)$$





Logic Gates: Sample 4

•Building smoke alarm system





Building Smoke Alarm System

• Components:

0	Two Active-low smoke detectors	$\overline{D1},\overline{D2}$
0	A sprinkler	
	 Active-low input to the sprinkler 	SPK
0	An automatic telephone dialer	$\overline{DI4I}$
	 Active-low input to the telephone dialer 	DIAL

• Behavior:

- Sprinkler is activated if either smoke detector detects smoke.
- When both smoke detector detect smoke, fire department is called.

 $\overline{SPK} = \overline{D1 + D2}$ $\overline{DIAL} = \overline{D1 \cdot D2}$


Building Smoke Alarm System





Electronic Logic Gates

Logic gates





Symbol set 2 (ANSI/IEEE Standard 91-1984)

OR



Logic Gates: Sample 1





Logic Gates: Sample 2











Thank You

