

# Digital Logic Design

---

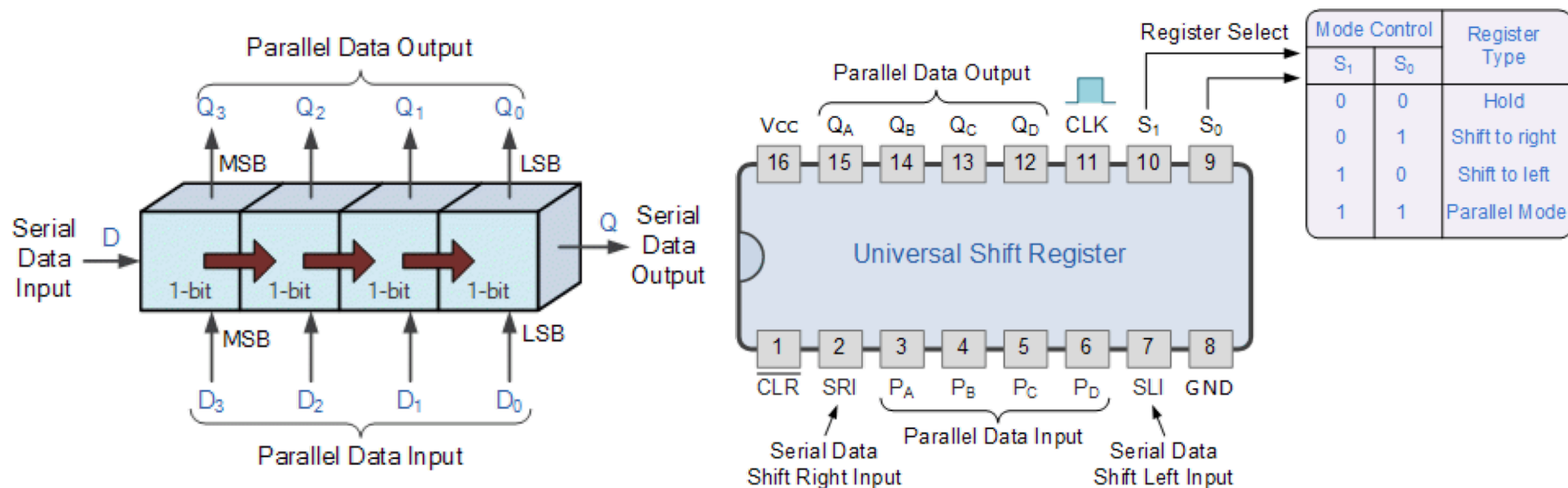
Hajar Falahati

Department of Computer Engineering  
IRAN University of Science and Technology

[hfalahati@iust.ac.ir](mailto:hfalahati@iust.ac.ir)

# Shift Register: Types

- Shift register
  - Parallel-in Parallel-out
  - Serial-in Serial-out
  - Serial-in Parallel-out
  - Parallel-in Serial-out



# Outline

---

- Counters



# Counters

---

# Counter

---

- A circuit that **cycles** through a **specified** number of **states**
- Applications
  - Counting!
  - Chronometer
  - Traffic light



# Counter: Type

---

- Synchronous
  - Applies the same clock to all flip flops
  - Parallel counter
  
- Asynchronous
  - Do not have a common clock pulse
  - A.k.a., ripple counter
    - Ripple clock pulse
    - Flip-flop outputs can be used as a **source of clock** for other flip-flops

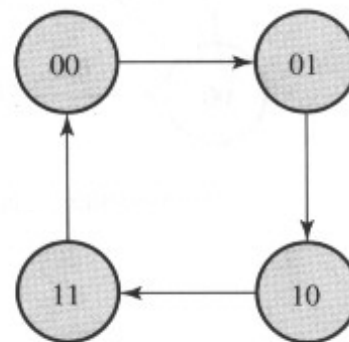
# Asynchronous Counters

---

# 2-bit Ripple Counter

---

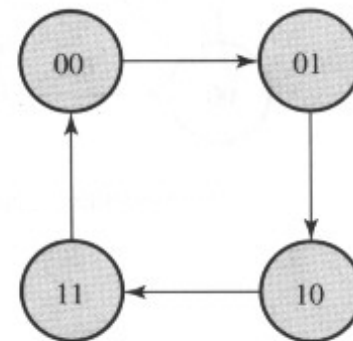
- Design a 2-bit ripple binary counter





# 2-bit Ripple Counter: Truth Table

- We need two output
  - $Q_0$  toggles every cycle
  - $Q_1$  toggles every two cycle

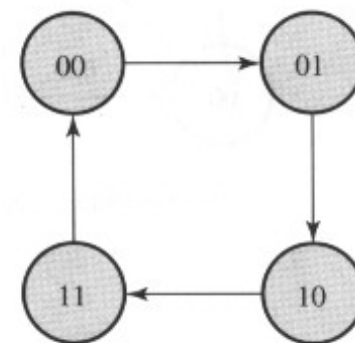


$Q_1$	$Q_0$
0	0
0	1
1	0
1	1

Recycle

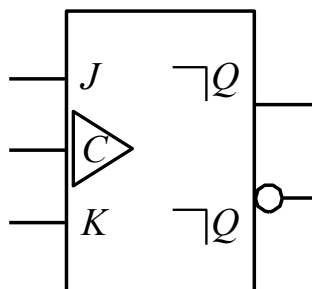
# 2-bit Ripple Counter: Flip Flops

- Flip flops candidates
  - JK



$Q_1$	$Q_0$
0	0
0	1
1	0
1	1

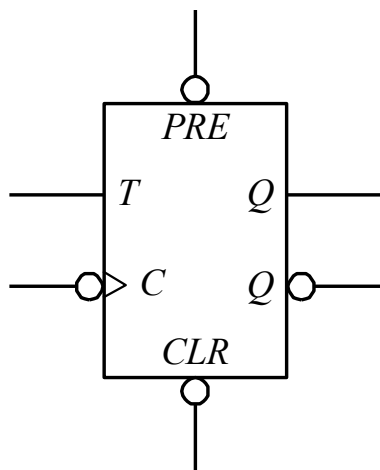
Recycle



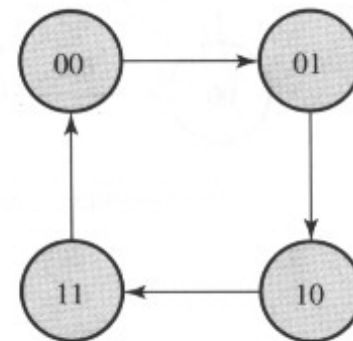
$J$	$K$	$Q$	$C$	$Q^*$
0	0	0		0 Hold
0	0	1		1
0	1	0		0 Reset
0	1	1		0
1	0	0		1 Set
1	0	1		1
1	1	0		1 Toggle
1	1	1		0

# 2-bit Ripple Counter: Flip Flops

- Flip flops candidates
  - $JK$
  - $T$



$T$	$Q$	$C$	$Q^*$
0	0	↓	0 Hold
0	1	↓	1
$\boxed{1}$	$\boxed{0}$	$\boxed{\downarrow}$	1 Toggle
1	1	↓	0

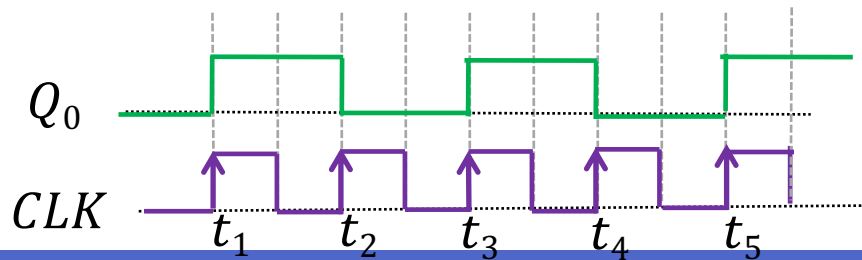
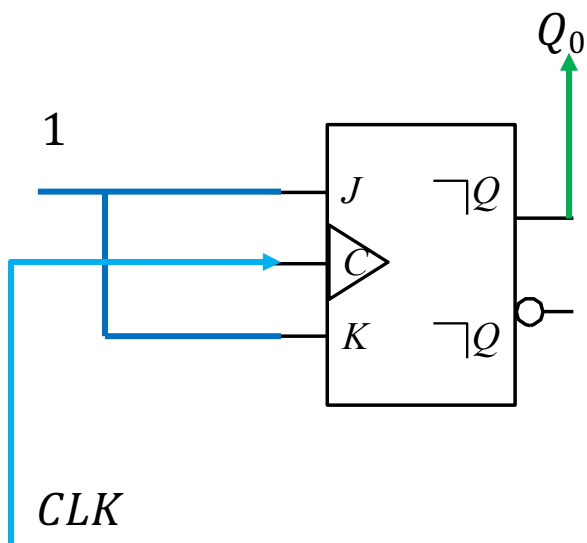
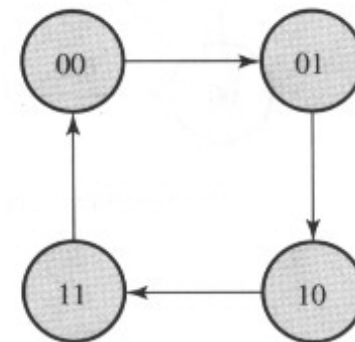


$Q_1$	$Q_0$
0	0
0	1
1	0
1	1

Recycle

# 2-bit Ripple Counter: JK FF

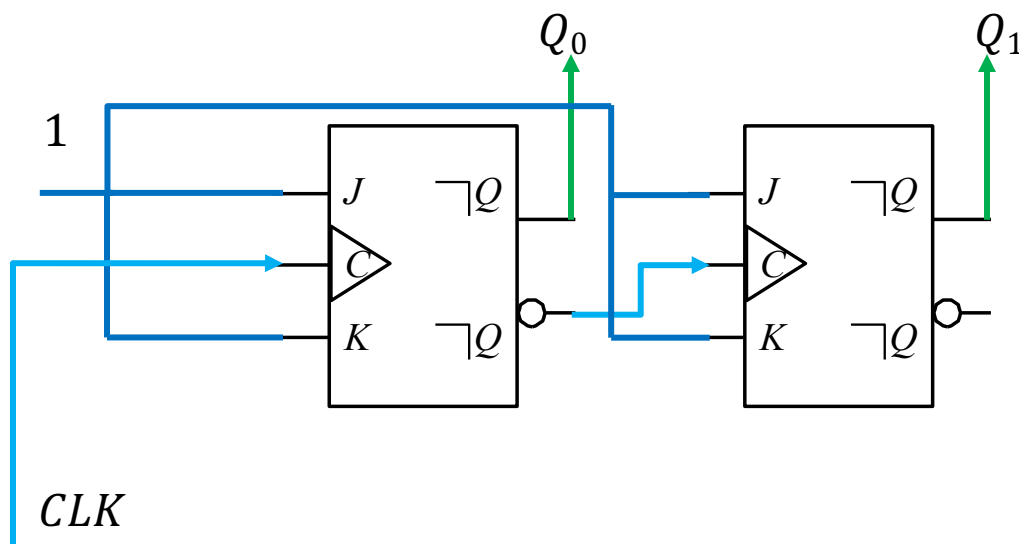
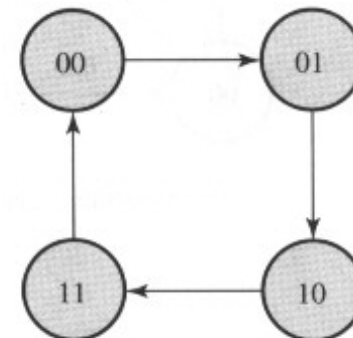
- Outputs
  - $Q_0$  toggles every cycle
  - $Q_1$  toggles every two cycle



CLK	$Q_0$
↑	0
↑	1
↑	0
↑	1

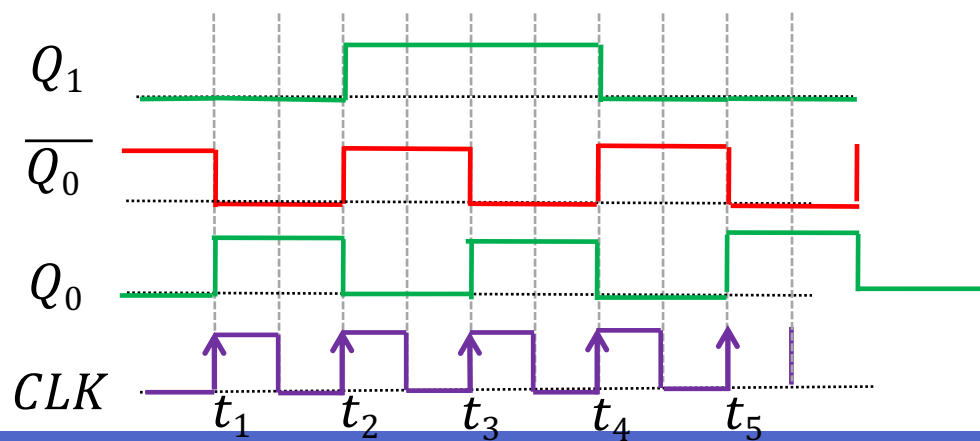
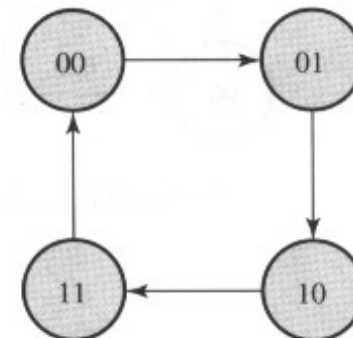
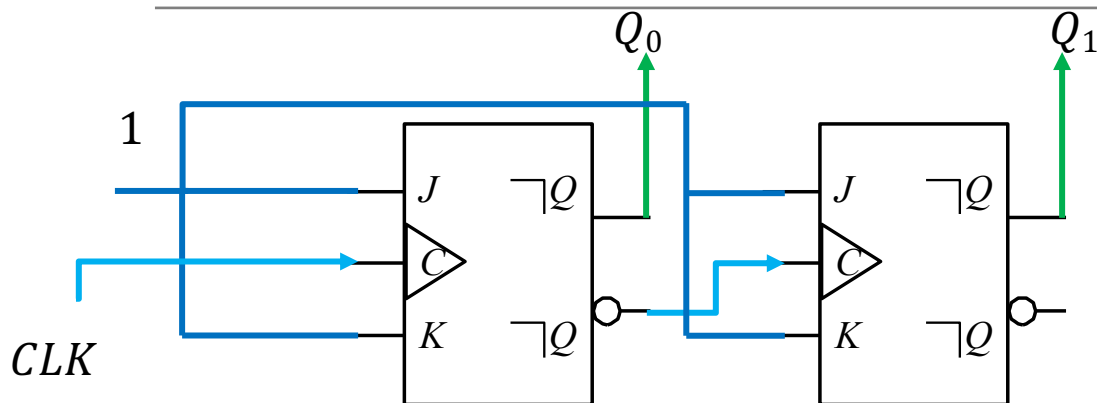
# 2-bit Ripple Counter: JK FF (cont'd)

- Outputs
  - $Q_0$  toggles every cycle
  - $Q_1$  toggles every two cycle



$CLK$	$Q_1$	$Q_0$	$\overline{Q_0}$
↑	0	0	1
↑	0	1	0
↑	1	0	1
↑	1	1	0

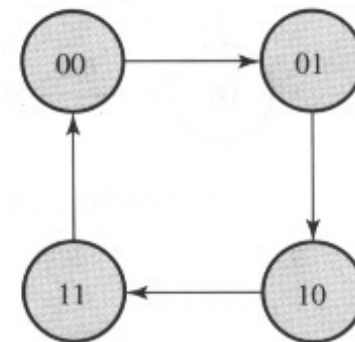
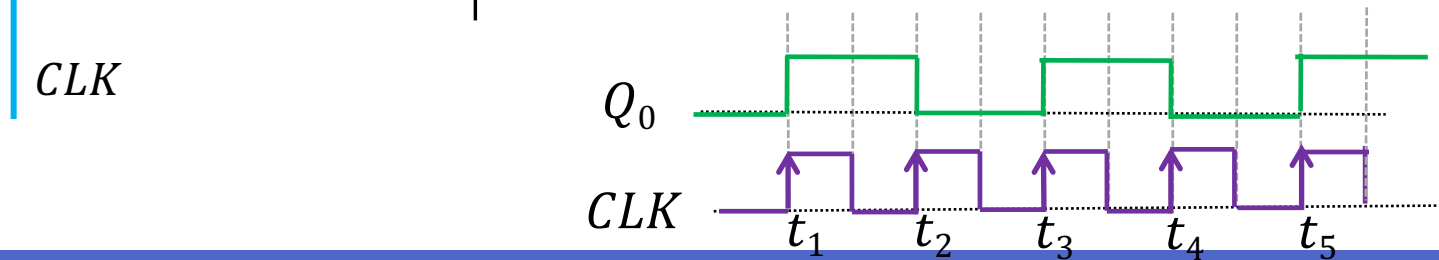
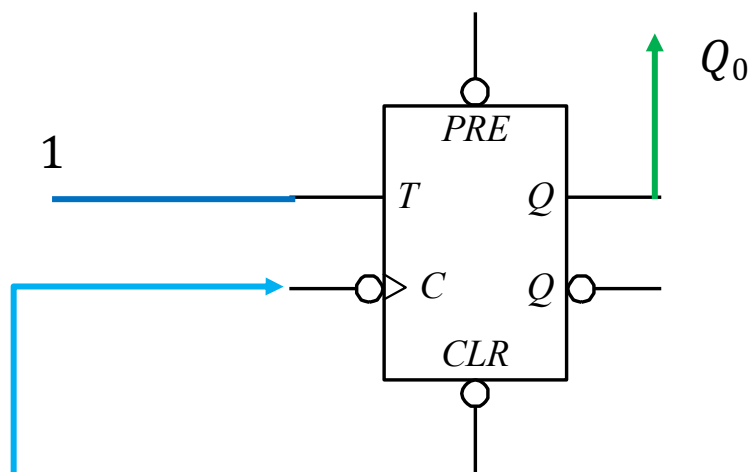
# 2-bit Ripple Counter: JK FF (cont'd)



$CLK$	$Q_1$	$Q_0$	$\overline{Q_0}$
↑	0	0	1
↑	0	1	0
↑	1	0	1
↑	1	1	0

# 2-bit Ripple Counter: T FF

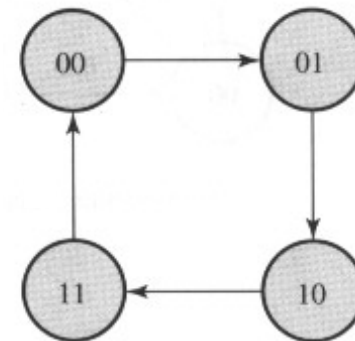
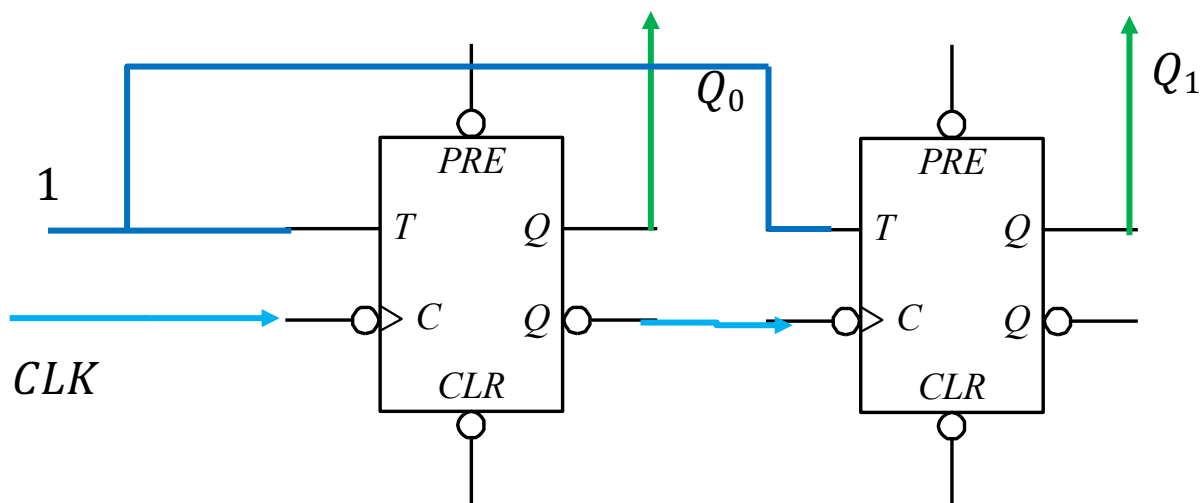
- Outputs
  - $Q_0$  toggles every cycle
  - $Q_1$  toggles every two cycle



CLK	$Q_0$
↑	0
↑	1
↑	0
↑	1

# 2-bit Ripple Counter: T FF

- Outputs
  - $Q_0$  toggles every cycle
  - $Q_1$  toggles every two cycle

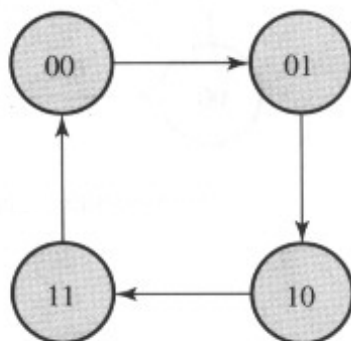


CLK	$Q_0$
↑	0
↑	1
↑	0
↑	1



# 2-bit Ripple Counter: D FF

- Flip flops candidates
  - $D$

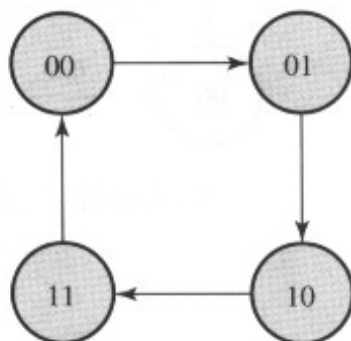


$Q_1$	$Q_0$
0	0
0	1
1	0
1	1

Recycle

# 2-bit Ripple Counter: D FF

- Flip flops candidates
  - $D$
  - Negative edge

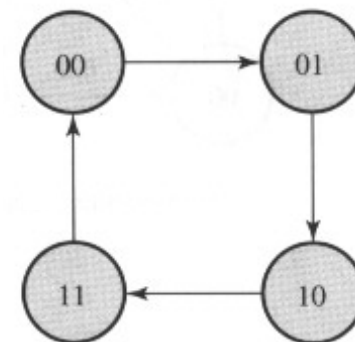
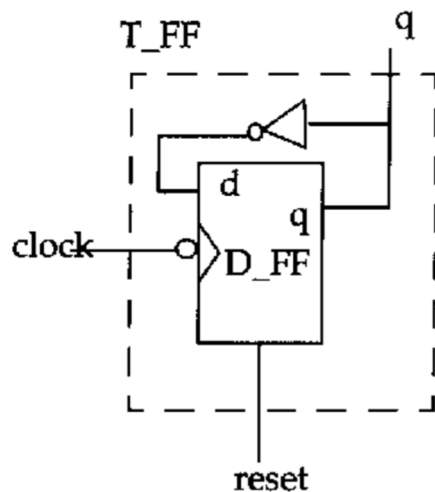


$Q_1$	$Q_0$
0	0
0	1
1	0
1	1

Recycle

# 2-bit Ripple Counter: D FF

- Flip flops candidates
  - $D$
  - Negative edge

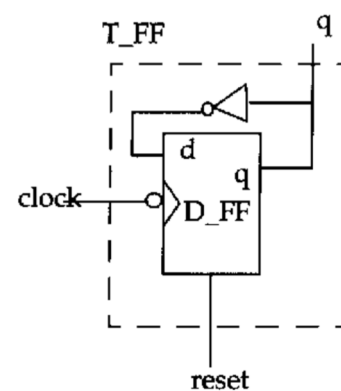
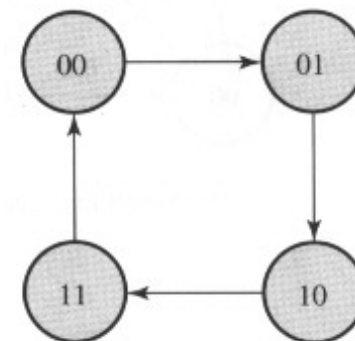
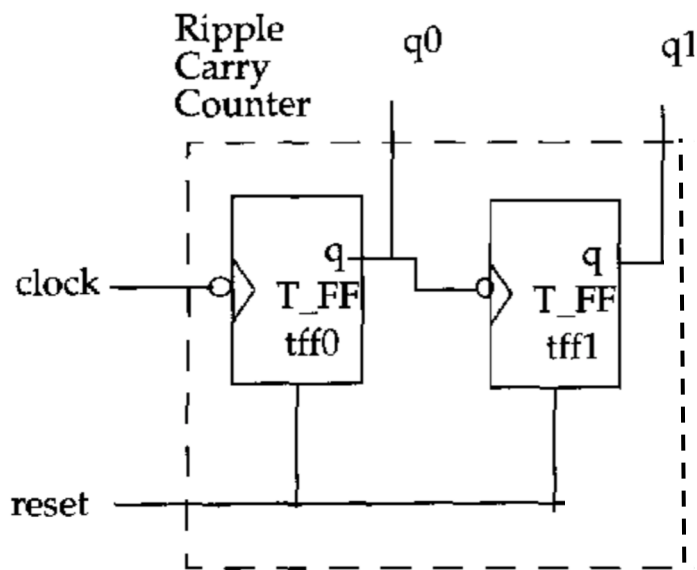


$Q_1$	$Q_0$
0	0
0	1
1	0
1	1

Recycle

# 2-bit Ripple Counter: D FF

- Flip flops candidates
  - $D$
  - Negative edge



$Q_1$	$Q_0$
0	0
0	1
1	0
1	1

Recycle

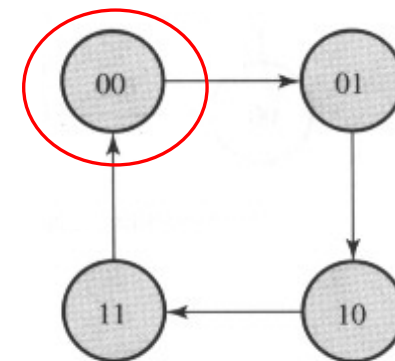
# Asynchronous Up/Down Counter

- Up counters

- Counts upward from zero to the maximum value

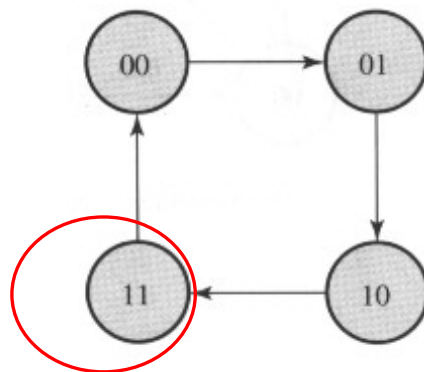
- Down counters

- Counts down from the maximum to zero value



$Q_1$	$Q_0$
1	1
1	0
0	1
0	0

Recycle

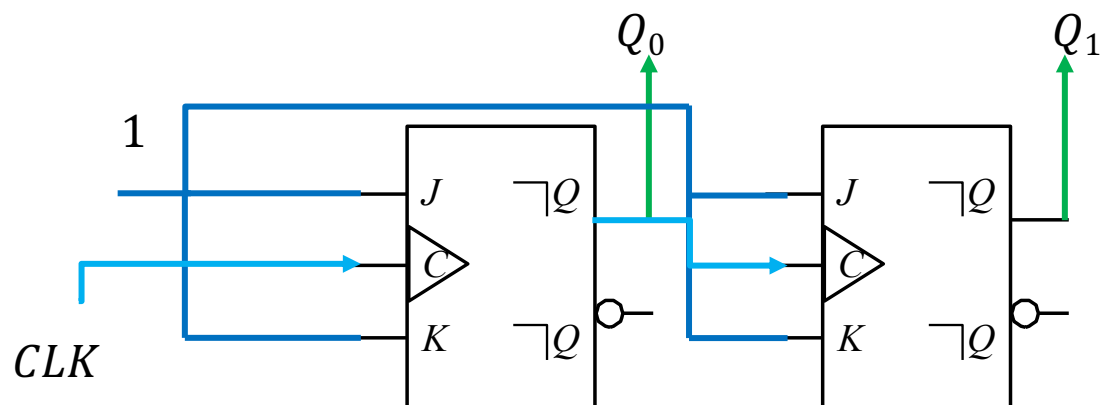


$Q_1$	$Q_0$
0	0
0	1
1	0
1	1

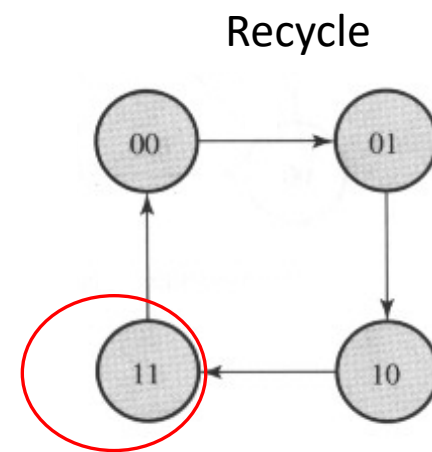
Recycle

# Asynchronous Up/Down Counter

- 2-bit down counter



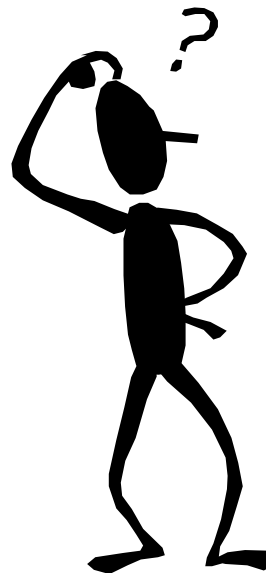
$Q_1$	$Q_0$
1	1
1	0
0	1
0	0



# Design an Asynchronous Counter

---

- Design an asynchronous 4-bit counter
  - Counting upward

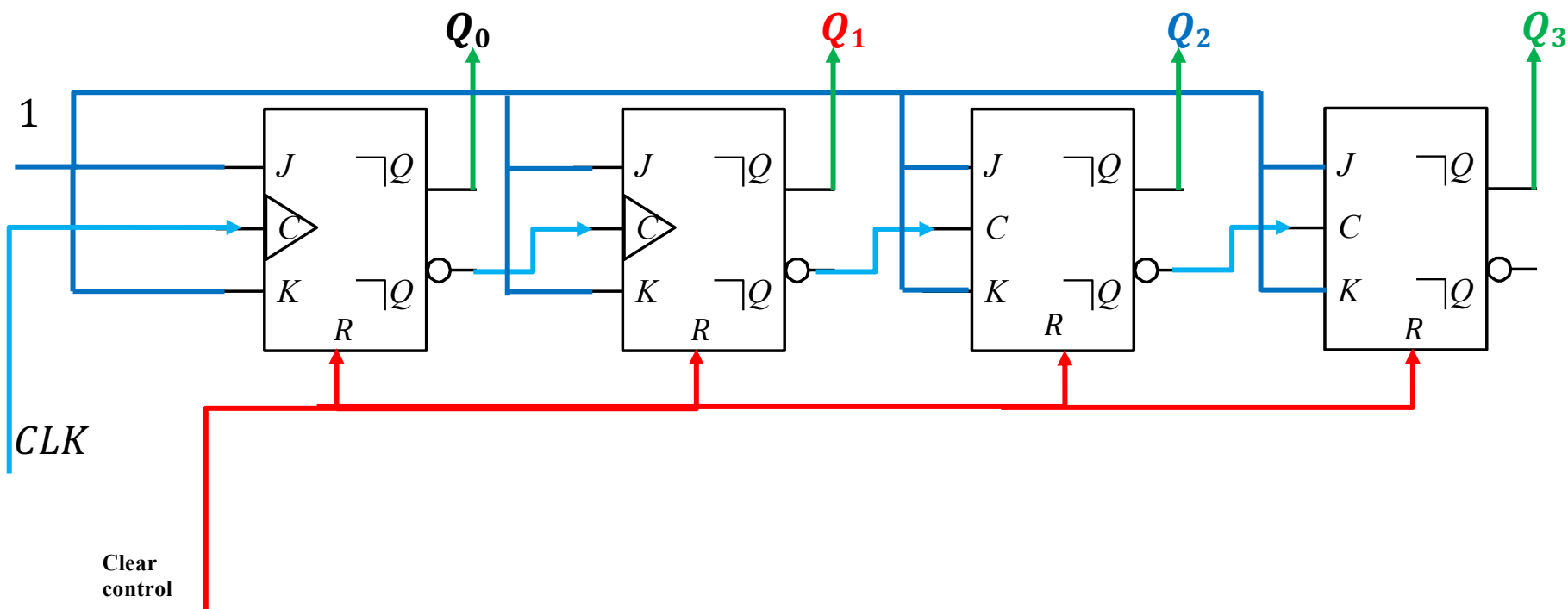


# Asynchronous 4-bit Counter

$Q_3$	$Q_2$	$Q_1$	$Q_0$				
0	0	0	0	←			
0	0	0	1	←			
0	0	1	0	←	←		
0	0	1	1	←			
0	1	0	0	←	←	←	
0	1	0	1	←			
0	1	1	0	←	←		
0	1	1	1	←			
1	0	0	0	←	←	←	←
1	0	0	1	←			
1	0	1	0	←	←		
1	0	1	1	←			
1	1	0	0	←	←	←	
1	1	0	1	←			
1	1	1	0	←	←		
1	1	1	1	←			

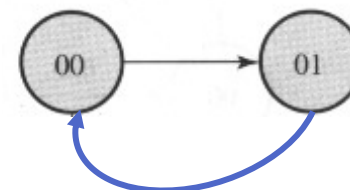
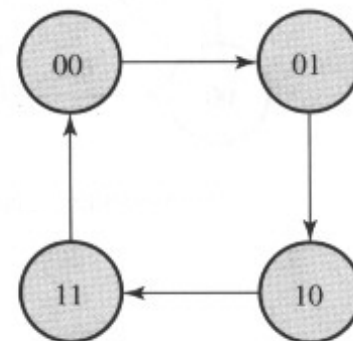


# Asynchronous 4-bit Counter



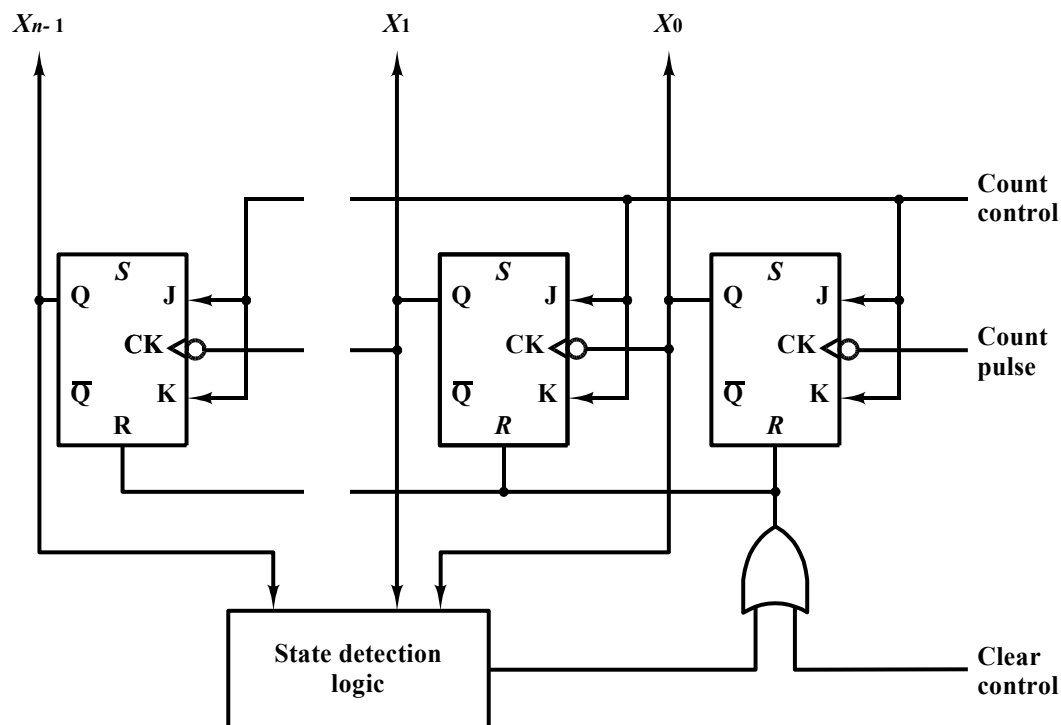
# Asynchronous Counter with Mod K

- **Mod- $2^n$  counters**
  - Counts through  $2^n$  states
  - n flip flops
- **Mod-x counters**
  - $x < 2^n$
  - Counts through x states
  - Truncated sequence



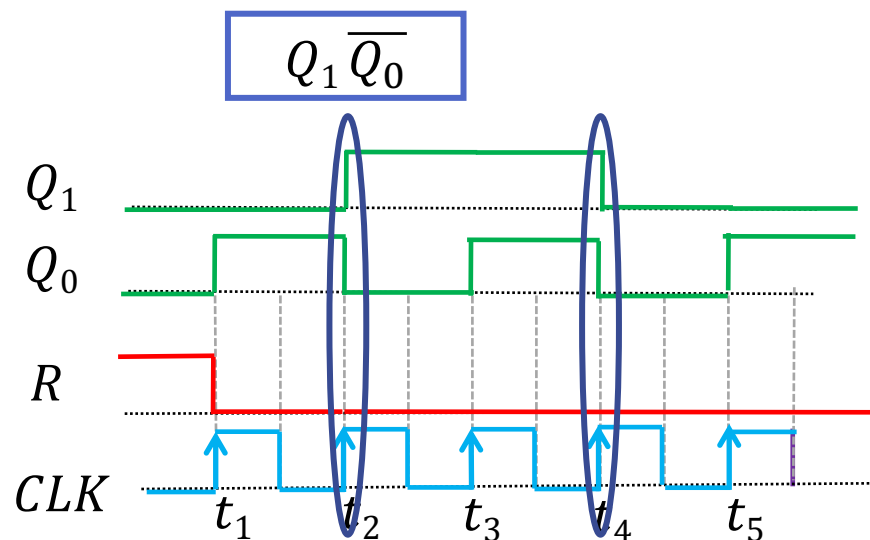
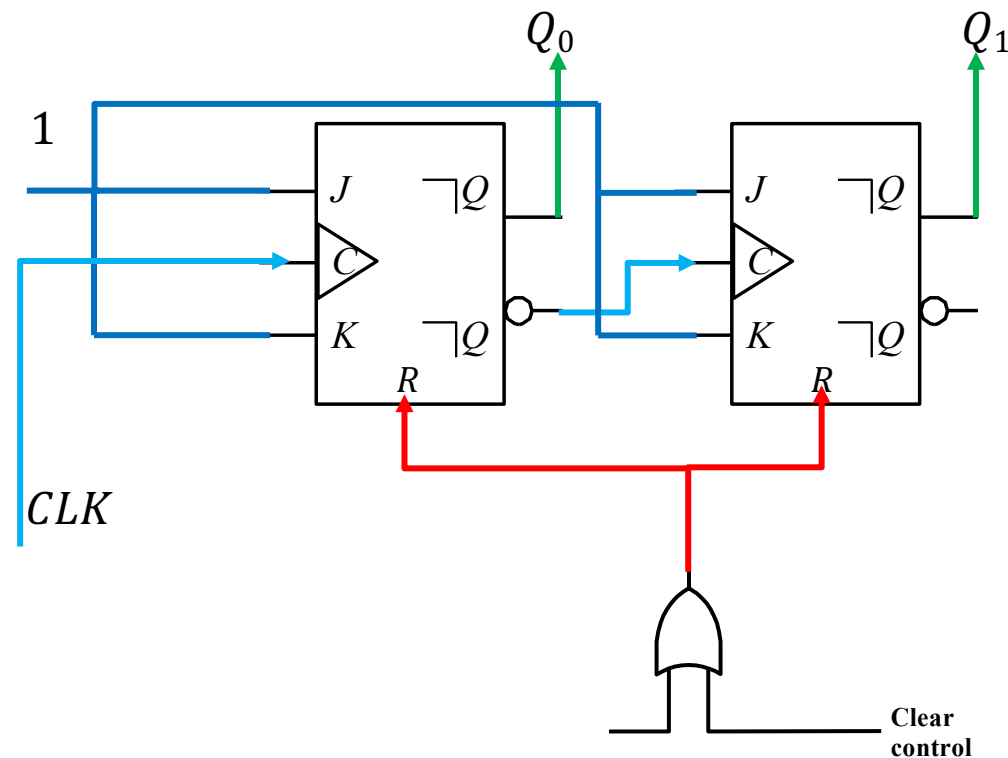
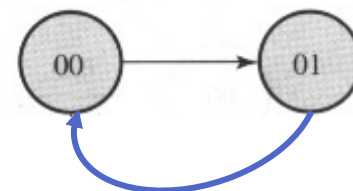
# Asynchronous Counter with Mod K (cont'd)

- How to design a Mod-x counters
  - **Recycle the** counter *at a special state*
  - **Not** traverse through *all* of the states in the binary sequence



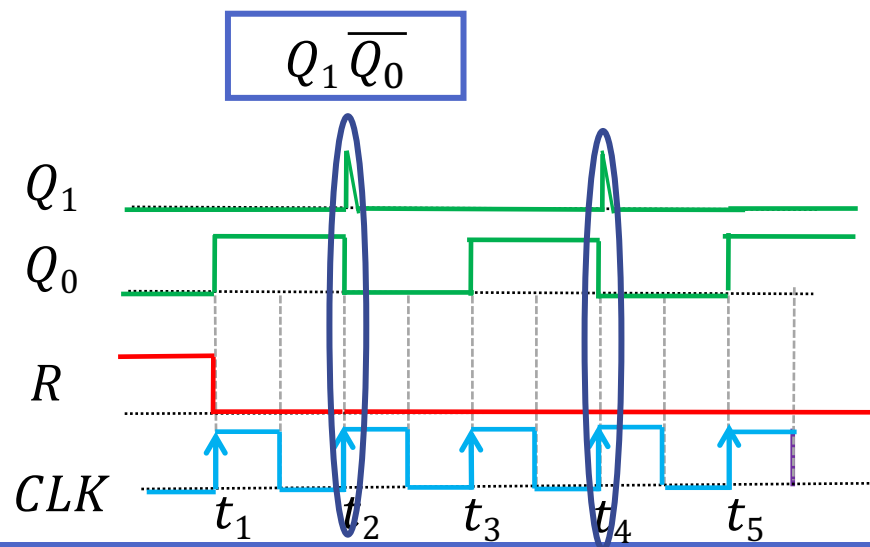
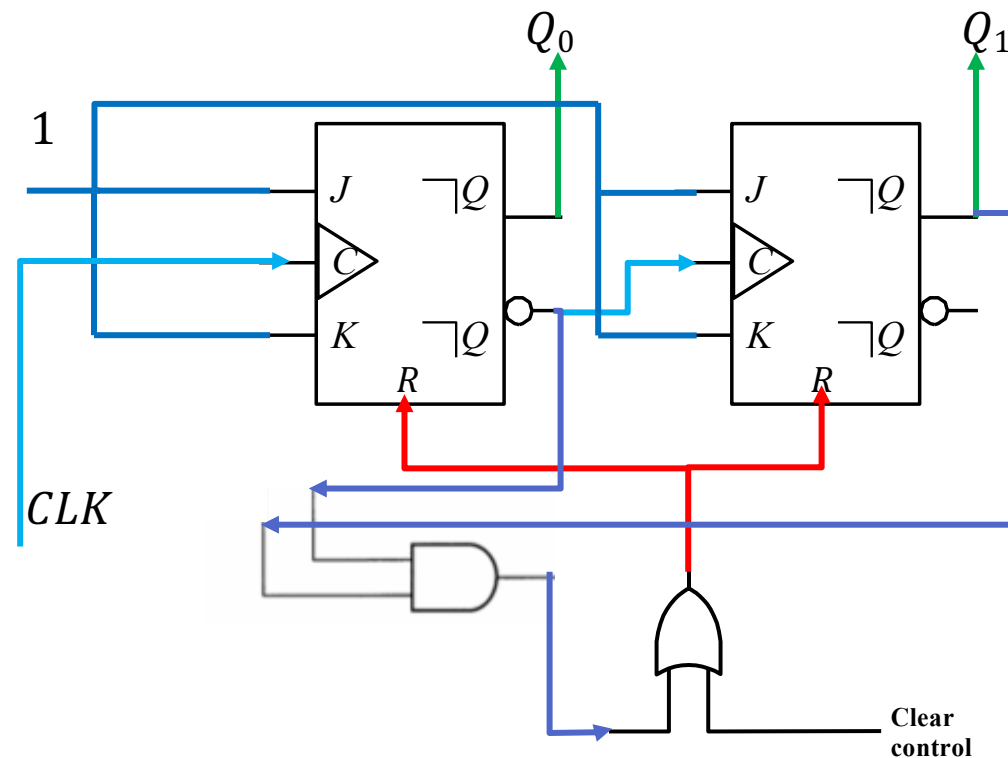
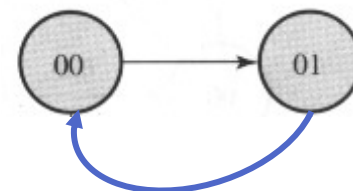
# Asynchronous Counter with Mod-2

- Example
  - Design a Mode-2 2-bit counter



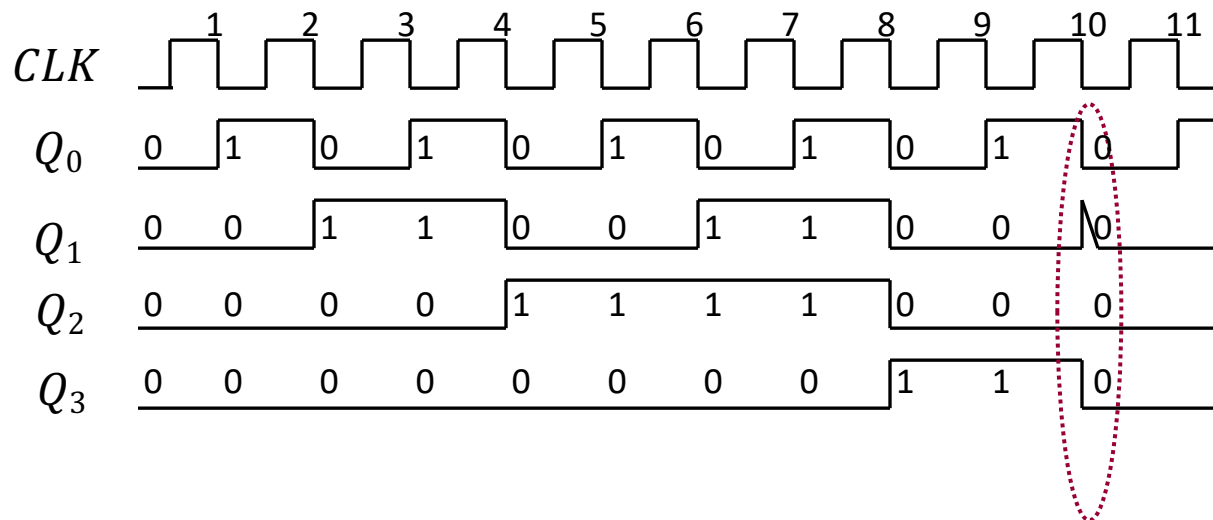
# Asynchronous Counter with Mod-2

- Example
  - Design a Mode-2 2-bit counter



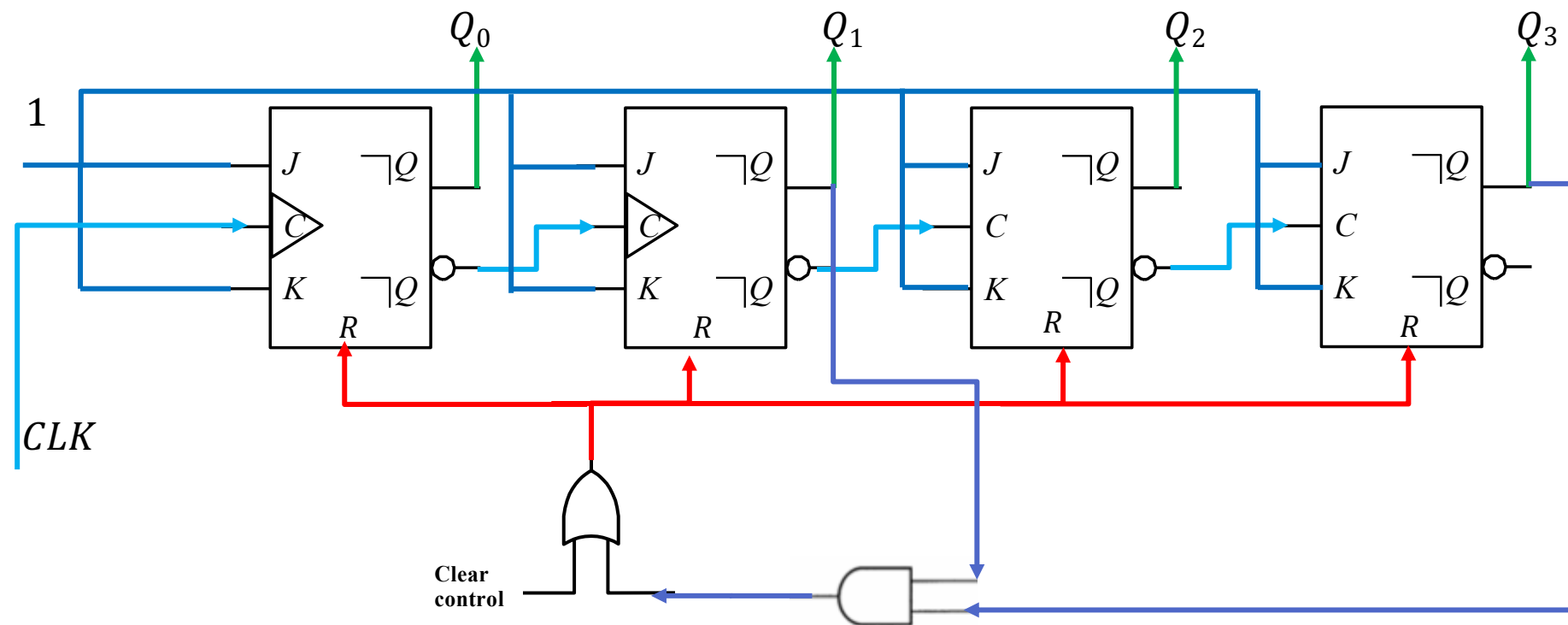
# Asynchronous Counter with Mod-10

- Decade counter
- BCD counter
  - Mode-10 4-bit counter



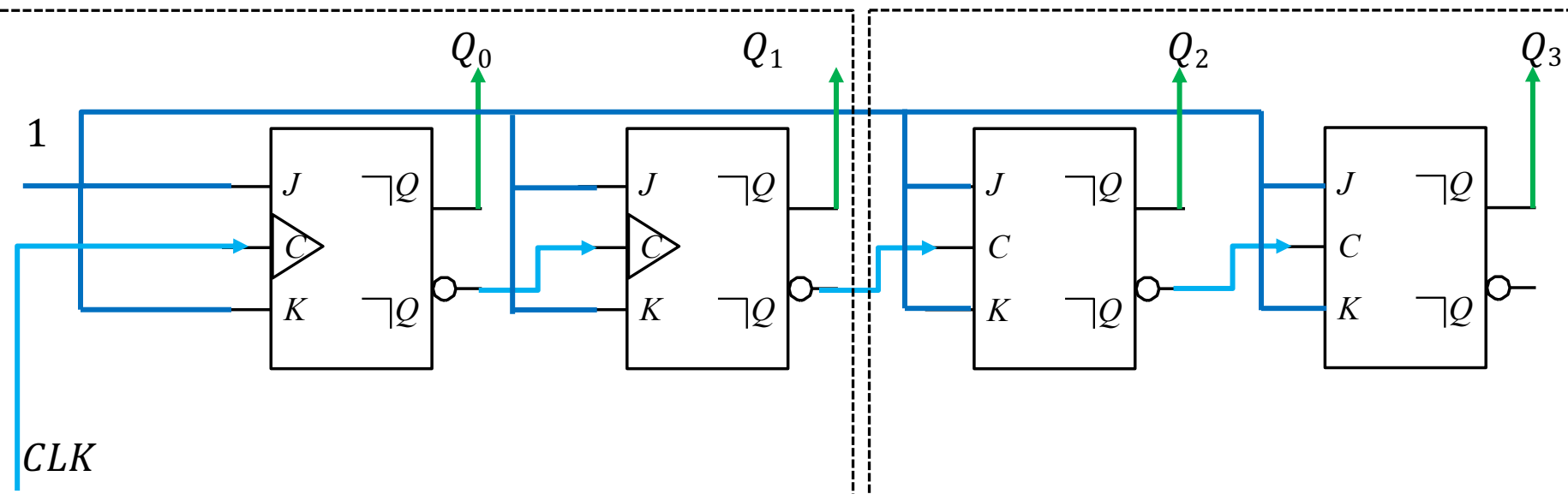
# Asynchronous Counter with Mod-10 (cont'd)

- Decade counter
- BCD counter
  - Mode-10 4-bit counter



# Asynchronous Counter: Cascading

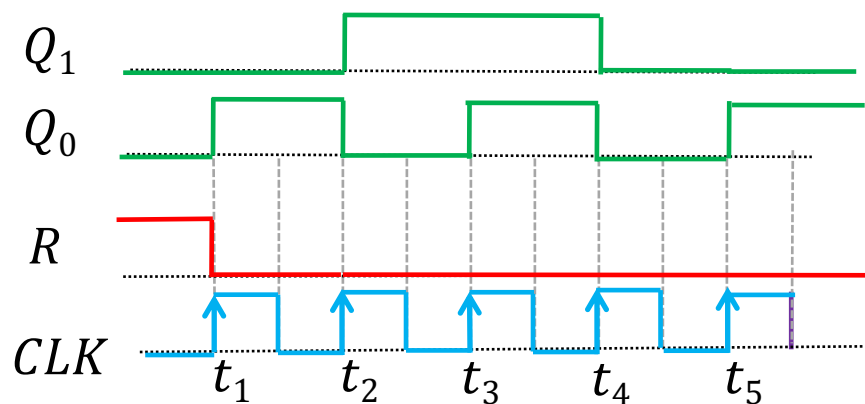
- Cascading smaller ripple counters -> Larger ripple counters
  - 2 2-bit counters → 4-bit counter





# Asynchronous Counter: Frequency Divider

- Frequency divider
  - Output of the last flip-flop (MSB) divides the input clock frequency by the MOD number of the counter

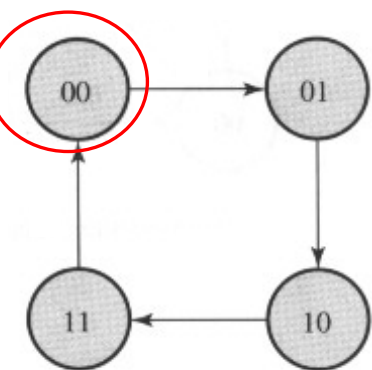


# Synchronous Counters

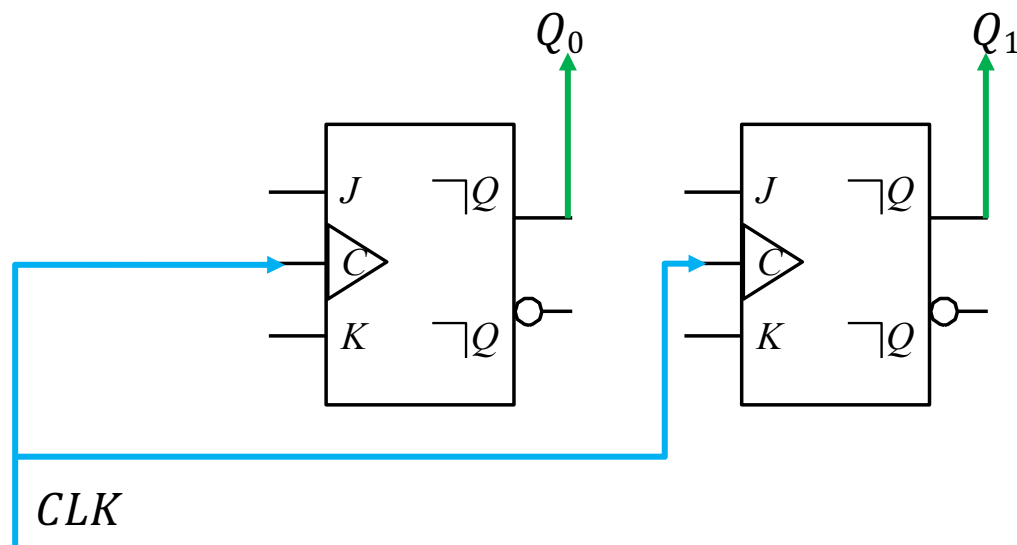
---

# Synchronous Counter

- Parallel counter
  - A common clock pulse
  - All flip flops are clocked at the same time
- Example: 2-bit synchronous binary counter
  - *J-K FFs*

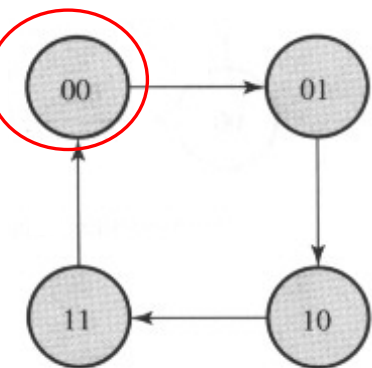


$Q_1$	$Q_0$
0	0
0	1
1	0
1	1

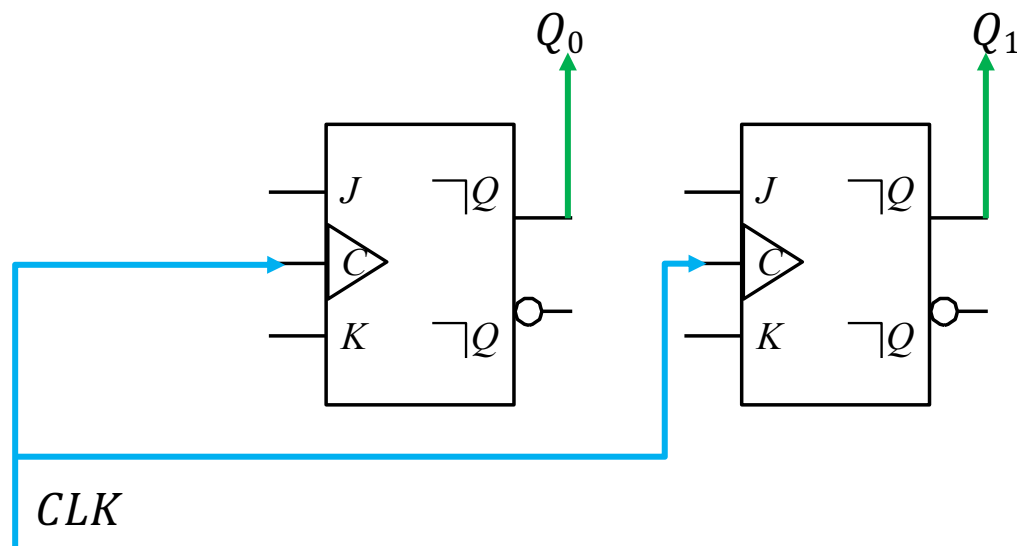


# 2-bit Synchronous Counter: JK-FF

- Example: 2-bit synchronous binary counter
  - J-K FFs
  - Determine flip flop inputs based on inputs and current states
  - Move from current state to the next one



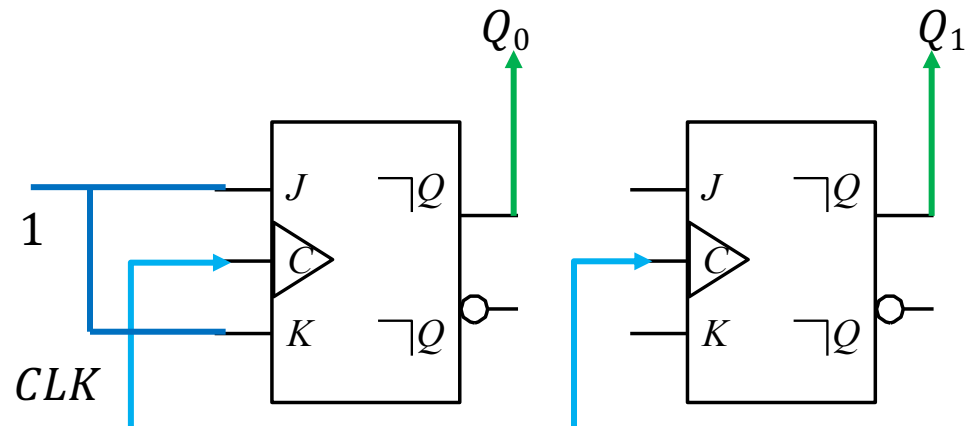
$Q_1$	$Q_0$
0	0
0	1
1	0
1	1



# 2-bit Synchronous Counter: JK-FF (cont'd)

Present State		Next State		FF0		FF1	
$Q_1$	$Q_0$	$Q_1$	$Q_0$	$J$	$K$	$J$	$K$
0	0	0	1	1	1		
0	1	1	0	1	1		
1	0	1	1	1	1		
1	1	0	0	1	1		

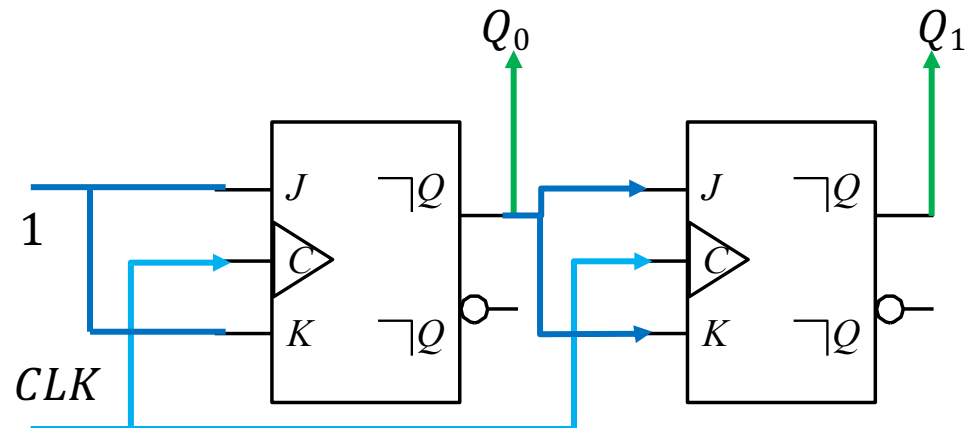
- FF0
  - $Q_0$  is toggling
  - FF should be in toggling state
  - $\Rightarrow J=K=1$



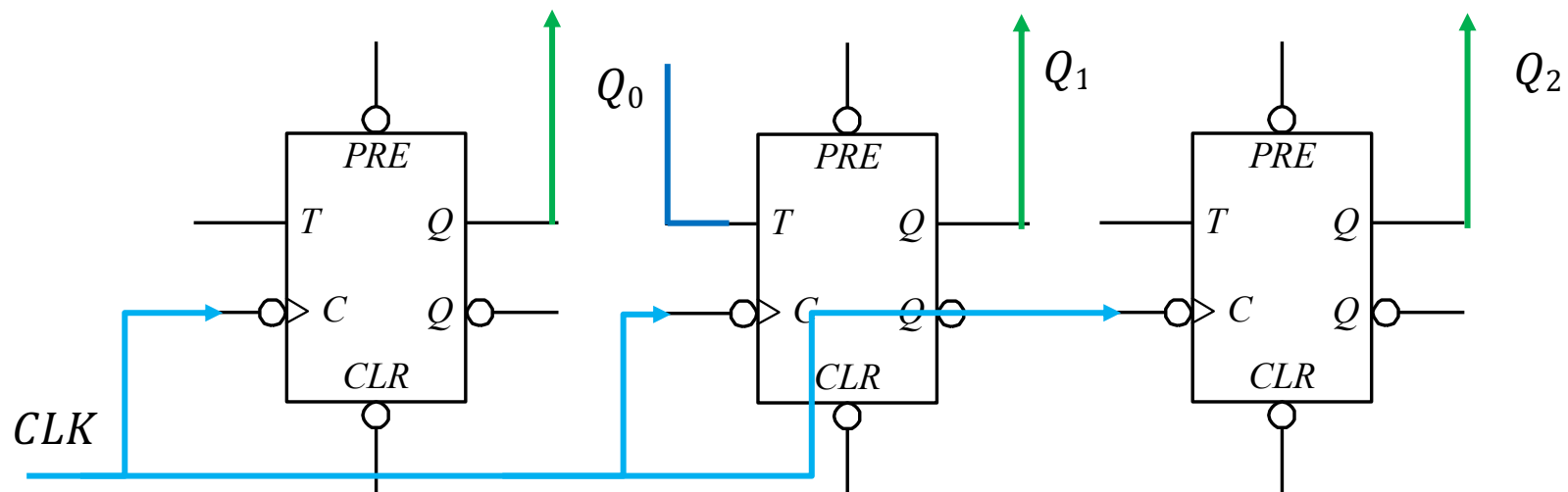
# 2-bit Synchronous Counter: JK-FF (cont'd)

Present State		Next State		FF0		FF1	
$Q_1$	$Q_0$	$Q_1$	$Q_0$	$J$	$K$	$J$	$K$
0	0	0	1	1	1	0	0
0	1	1	0	1	1	1	1
1	0	1	1	1	1	0	0
1	1	0	0	1	1	1	1

- FF1
  - $J = K = Q_0$



# 3-bit Synchronous Counter: T-FF



# 3-bit Synchronous Counter: T-FF (cont'd)

Present State			Next State			Flip Flops		
$Q_2$	$Q_1$	$Q_0$	$Q_2$	$Q_1$	$Q_0$	$T_2$	$T_1$	$T_0$
0	0	0	0	0	1			1
0	0	1	0	1	0			1
0	1	0	0	1	1			1
0	1	1	1	0	0			1
1	0	0	1	0	1			1
1	0	1	1	1	0			1
1	1	0	1	1	1			1
1	1	1	0	0	0			1



# 3-bit Synchronous Counter: T-FF (cont'd)

Present State			Next State			Flip Flops		
$Q_2$	$Q_1$	$Q_0$	$Q_2$	$Q_1$	$Q_0$	$T_2$	$T_1$	$T_0$
0	0	0	0	0	1		0	1
0	0	1	0	1	0		1	1
0	1	0	0	1	1		0	1
0	1	1	1	0	0		1	1
1	0	0	1	0	1		0	1
1	0	1	1	1	0		1	1
1	1	0	1	1	1		0	1
1	1	1	0	0	0		1	1

# 3-bit Synchronous Counter: T-FF (cont'd)

Present State			Next State			Flip Flops		
$Q_2$	$Q_1$	$Q_0$	$Q_2$	$Q_1$	$Q_0$	$T_2$	$T_1$	$T_0$
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1

$$T_0 = 1$$

$$T_1 = Q_0$$

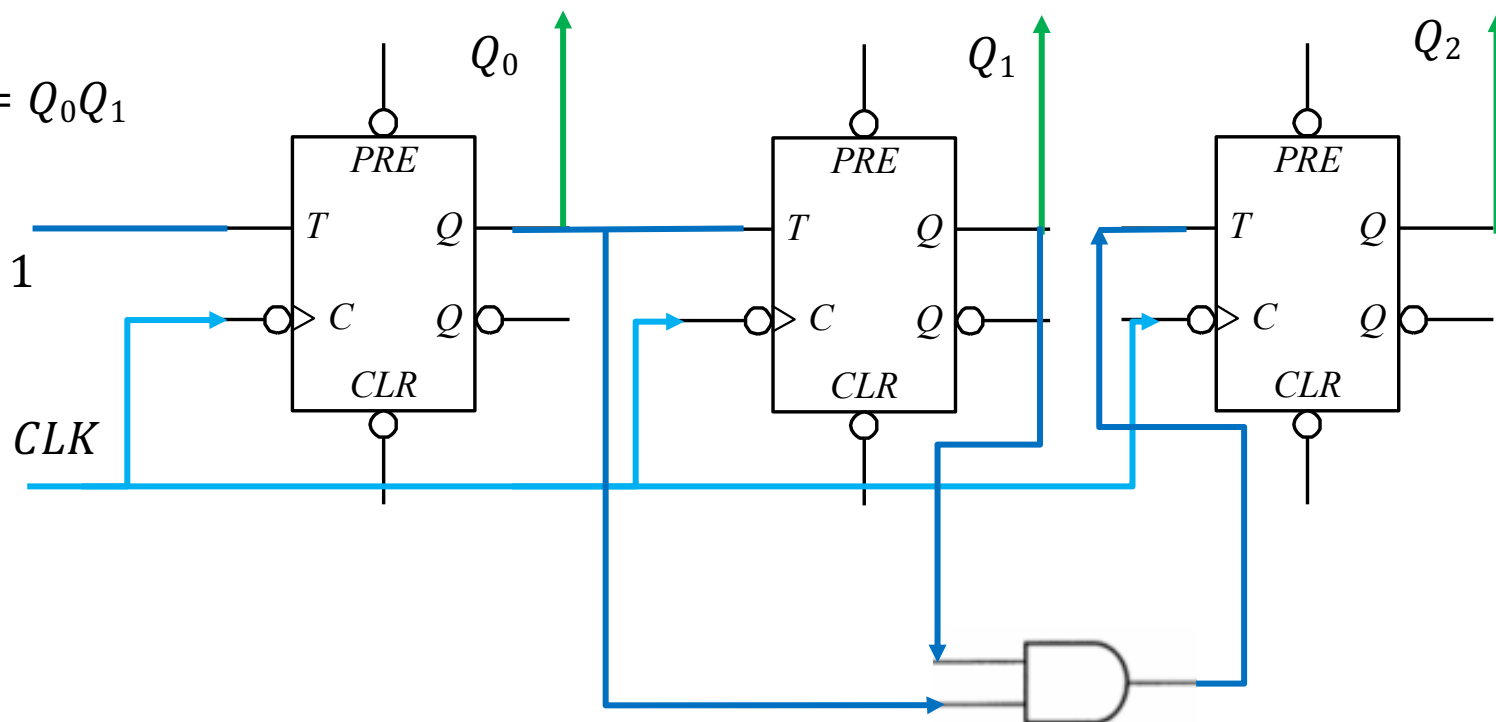
$$T_2 = Q_0Q_1$$

# 3-bit Synchronous Counter: T-FF (cont'd)

$$T_0 = 1 \quad T_n = Q_0 Q_1 \dots Q_{n-1}$$

$$T_1 = Q_0$$

$$T_2 = Q_0 Q_1$$



# Synchronous BCD Counter: T-FF

Present State				Next State				Flip Flops			
$Q_3$	$Q_2$	$Q_1$	$Q_0$	$Q_3$	$Q_2$	$Q_1$	$Q_0$	$T_3$	$T_2$	$T_1$	$T_0$
0	0	0	0	0	0	0	1	0	0	0	1
0	0	0	1	0	0	1	0	0	0	1	1
0	0	1	0	0	0	1	1	0	0	0	1
0	0	1	1	0	1	0	0	0	1	1	1
0	1	0	0	0	1	0	1	0	0	0	1
0	1	0	1	0	1	1	0	0	0	1	1
0	1	1	0	0	1	1	1	0	0	0	1
0	1	1	1	1	0	0	0	1	1	1	1
1	0	0	0	1	0	0	1	0	0	0	1
1	0	0	1	0	0	0	0	1	0	0	1

$$T_0 = 1$$

$$T_1 = \overline{Q_3}Q_0$$

$$T_2 = Q_0Q_1$$

$$T_3 = Q_0Q_1Q_2 + Q_0Q_3$$

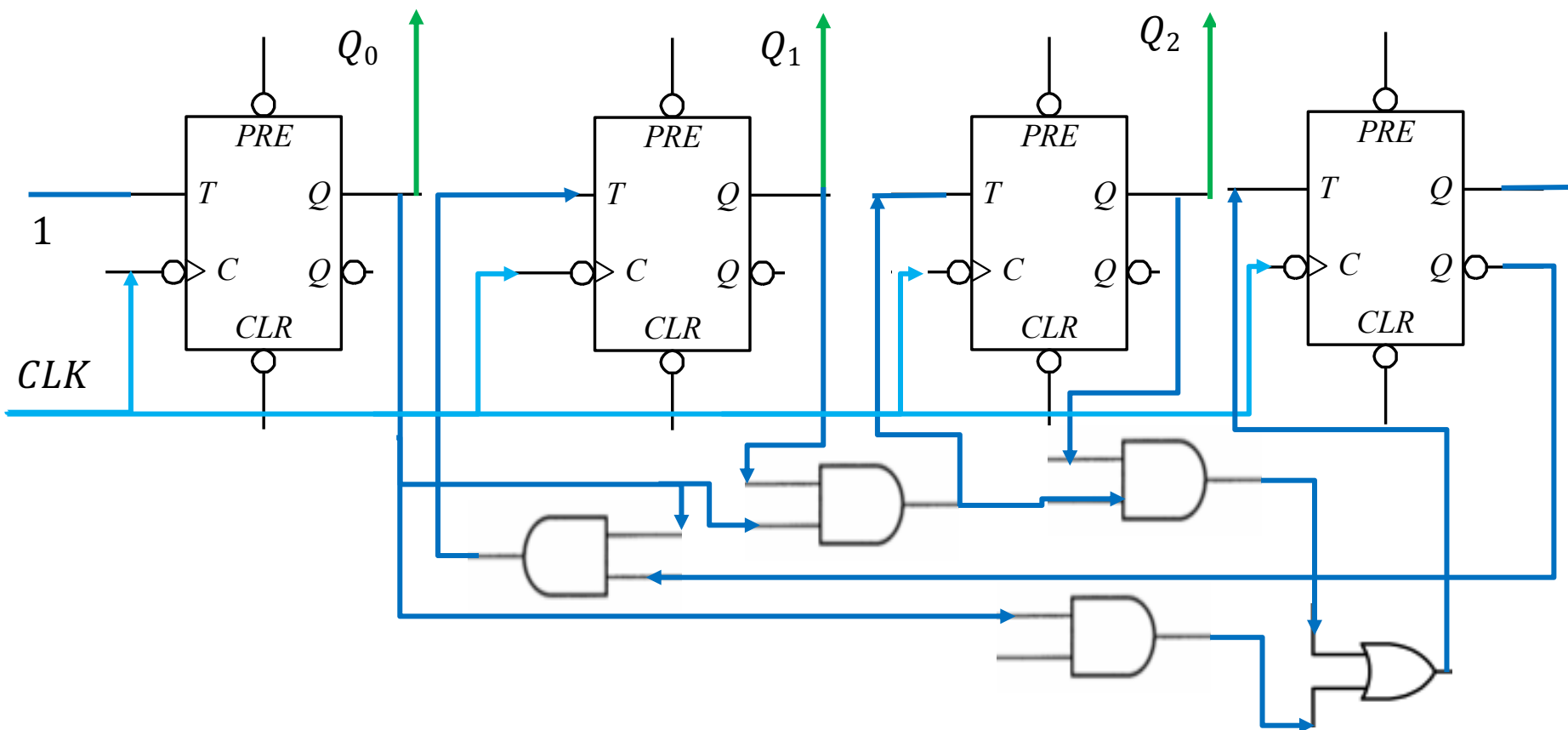
# Synchronous BCD Counter: T-FF(cont'd)

$$T_0 = 1$$

$$T_2 = Q_0Q_1$$

$$T_1 = \overline{Q_3}Q_0$$

$$T_3 = Q_0Q_1Q_2 + Q_0Q_3$$



# Design Sample

---

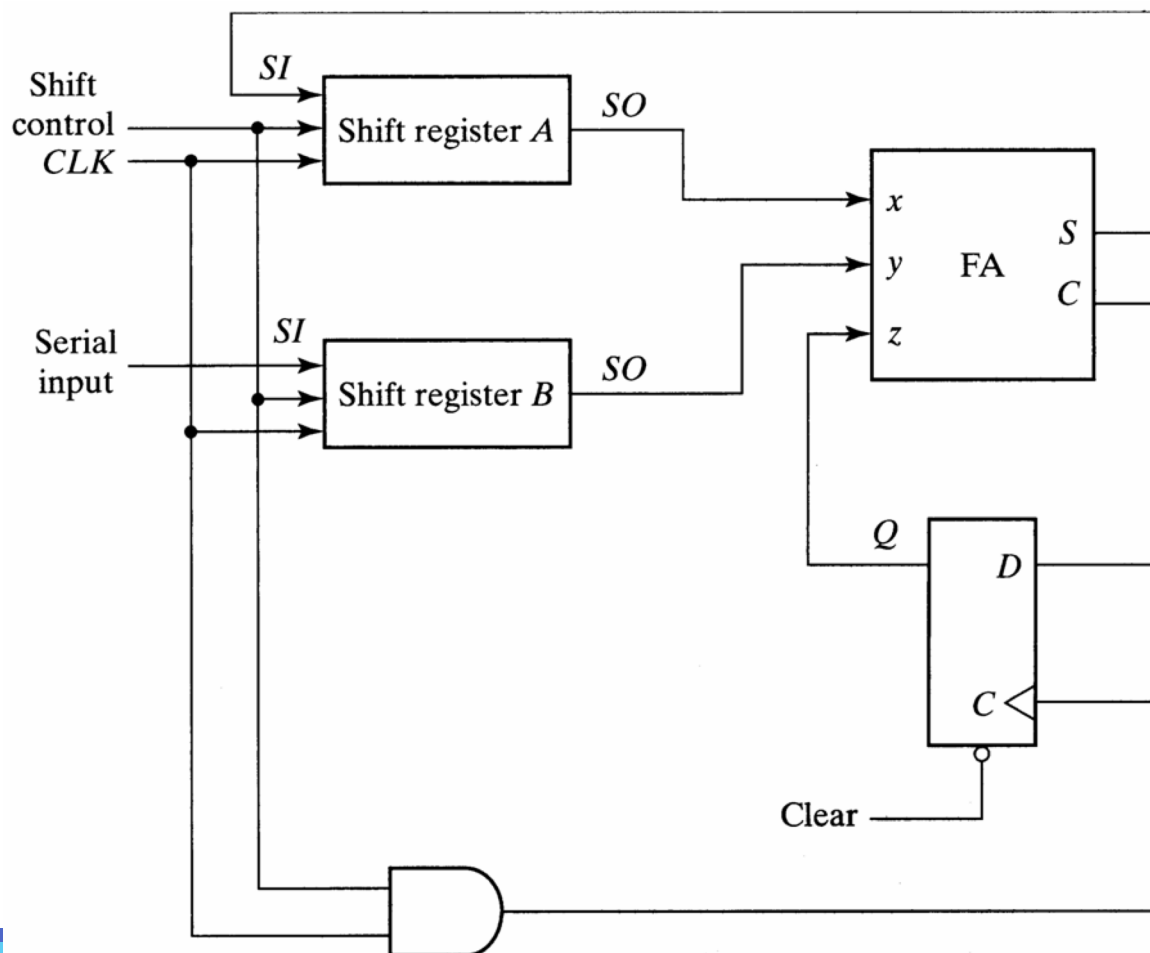
# Sample 1

---

- Design an adder using flip flops.

# Sample 1 (cont'd)

- Serial adder





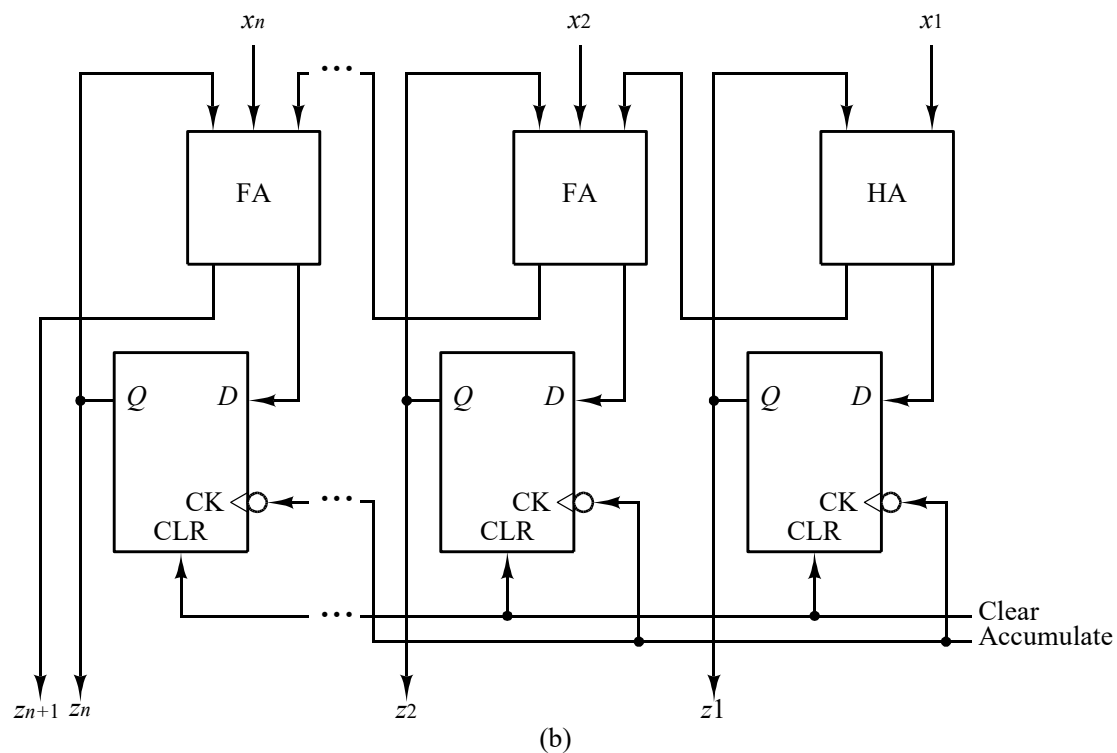
# Sample 2

---

- Design an accumulator using flip flips.
- An adder that totals a series of binary data

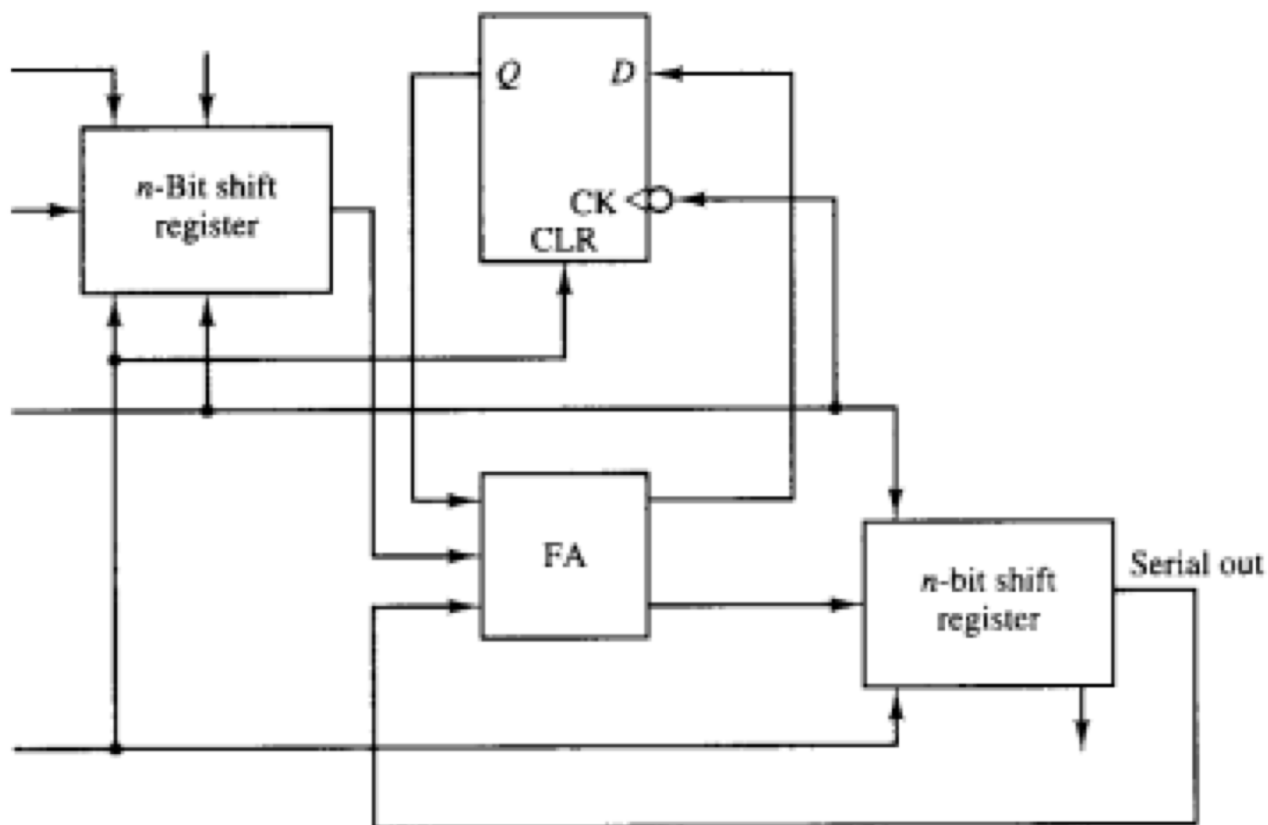
# Sample 2 (cont'd)

- Parallel Accumulator using flip flips



# Sample 2 (cont'd)

- Serial accumulator using flip flips



# Thank You

---

