Digital Logic Design

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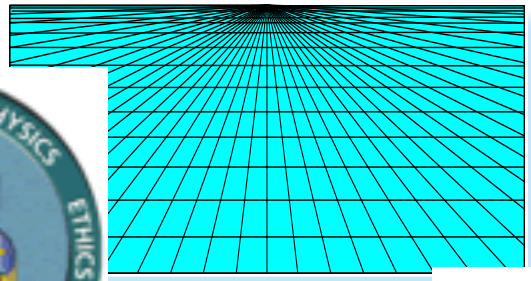
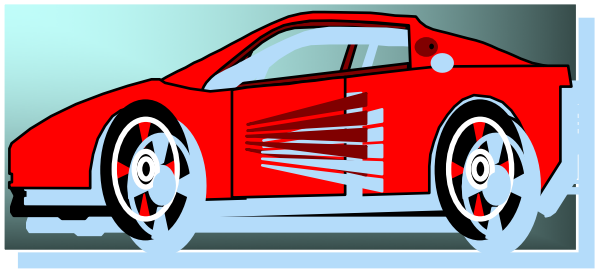
Digital Computer Systems Are Every Where!

Life Sciences



Aerospace

Internet & Ecommerce



CAD/CAM

Digital Biology

Military Applications

Outline

- Number Systems

Number Systems

Number System

- Positioned number

$$N = (a_{n-1}a_{n-2} \ldots a_1a_0 \,.\, a_{-1}a_{-2} \ldots a_{-m})_r$$

  - . = radix point
  - r = radix or base
  - n = number of integer digits to the left of the radix point
  - m = number of fractional digits to the right of the radix point
  - a_{n-1} = most significant digit (MSD)
  - a_{-m} = least significant digit (LSD)

- Polynomial notation
  - Series representation

$$N = a_{n-1} \times r^{n-1} + a_{n-2} \times r^{n-2} + \ldots + a_0 \times r^0 + a_{-1} \times r^{-1} \ldots + a_{-m} \times r^{-m}$$

$$\sum_{i=-m}^{n-1} a_i r^i$$

Number System: Some Bases

- **Decimal** numbers
  - Digits= {0,1,2,3,4,5,6,7,8,9}
  - $(251.41)_{10} = 2 \times 10^2 + 5 \times 10^1 + 1 \times 10^0 + 4 \times 10^{-1} + 1 \times 10^{-2}$
- **Binary** numbers
  - Digits = {0, 1}
  - $(11010.11)_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} = (26.75)_{10}$
  - 1 K (kilo) = 2^{10} = 1,024
  - 1M (mega) = 2^{20} = 1,048,576,
  - 1G (giga) = 2^{30} = 1,073,741,824
- **Octal** numbers
  - Digits = {0, 1, 2, 3, 4, 5, 6, 7}
  - $(127.4)_8 = 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1} = (87.5)_{10}$
- **Hexadecimal** numbers
  - Digits = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}
  - $(1A.A)_{16} = 1 \times 16^1 + A \times 16^0 + A \times 16^{-1} = (26.625)_{10}$

Number System: Important Numbers

Decimal	Binary	Octal	Hexadecimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	1	F
16	10000	20	10

Number-Base Conversion

- **Representations** of a number in a **different radix** are **equivalent**
  - If they have the **same decimal representation**
  - $(0011)_8 = (1001)_2$
    - Both have decimal value **9**

- Conversion from base ***r*** to ***decimal***
  - Expanding the number in a power series
  - Adding all the terms

- Conversion from ***decimal*** to base ***r***
  - Radix divide method
  - Conversion Procedure

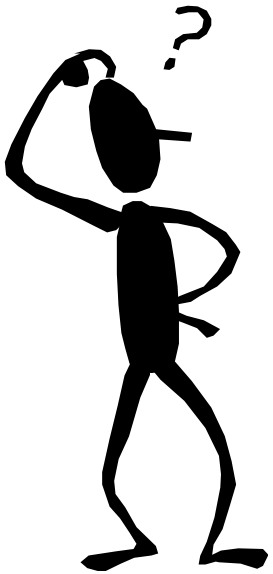
Binary to Decimal

- $\sum_{i=-m}^{n-1} a_i 2^i$
- $(1010)_2$ = 10

Digit notation	a_3	a_2	a_1	a_0
Digit	1	0	1	0
Value	2^3	2^2	2^1	2^0
Results = Value * Digit	8	0	2	0

Binary to Decimal: Sample 1

- $(11011)_2$
- $(100111011)_2$ = ?
- $(111010010)_2$ = ?



Binary to Decimal: Sample 1

- $(11011)_2$ = 27
- $(100111011)_2$ = 315
- $(111010010)_2$ = 466

Digit notation	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0
Digit	0	0	0	0	1	1	0	1	1
	1	0	0	1	1	1	0	1	1
	1	1	1	0	1	0	0	1	0
Value	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Results = Value * Digit	0	0	0	0	16	8	0	2	1
	256	0	0	32	16	8	0	2	1
	256	128	64	0	16	0	0	2	0

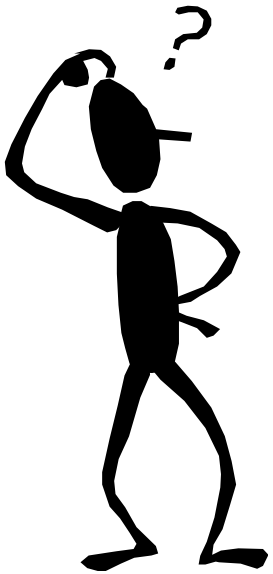
Binary to Decimal: Fraction

- $\sum_{i=-m}^{n-1} a_i 2^i$
- Convert the integer part
- Convert the fractional part
- Join them with radix point
- $(1010.11)_2$ = (8 + 2). (0.5 + 0.25) = 10.75

Digit notation	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}
Digit	1	0	1	0	1	1
Value	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}
Results = Value * Digit	8	0	2	0	0.5	0.25

Binary to Decimal: Fraction Sample 2

- $(100111011.01)_2$ = ?
- $(111010010.10)_2$ = ?



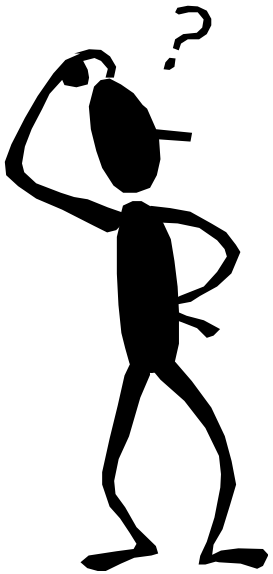
Binary to Decimal: Sample 2

- $(100111011.01)_2$ = 315.***25***
- $(111010010.10)_2$ = 466.***5***

Digit notation	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}
Digit	1	0	0	1	1	1	0	1	1	0	1
	1	1	1	0	1	0	0	1	0	1	0
Value	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}
Results = Value * Digit	256	0	0	32	16	8	0	2	1	**0**	***0.25***
	256	128	64	0	16	0	0	2	0	***0.5***	0

Conversion to Decimal: Sample 3

- $(1010)_8$ = (?) $_{10}$
- $(5062.42)_8$ = (?) $_{10}$



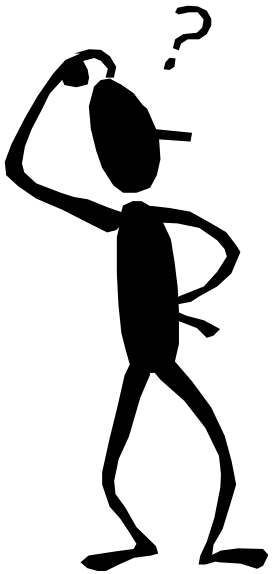
Binary to Decimal: Sample 3

- $(1010)_8 = (520.\mathbf{00})_{10}$
- $(5062.42)_8 = (2610.\mathbf{53125})_{10}$

Digit notation	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}
Digit	0	0	0	0	0	1	0	1	0	0	0
	0	0	0	0	0	5	0	6	2	4	2
Value	8^8	8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	8^{-1}	8^{-2}
Results = Value * Digit	0	0	0	0	0	512	0	8	0	**0**	***0***
	0	0	0	0	0	2560	0	48	2	***0.5***	**0.03125**

Conversion to Decimal: Sample 4

- $(1AB0)_{16} = (?)_{10}$
- $(1AB0.A)_{16} = (?)_{10}$



Binary to Decimal: Sample 4

- $(1AB0)_{16}$ = $(6832.\mathbf{00})_{10}$
- $(1AB0.A)_{16}$ = $(6832.\mathbf{625})_{10}$

Digit notation	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0	a_{-1}	a_{-2}
Digit	0	0	0	0	0	1	A	B	0	0	0
	0	0	0	0	0	1	A	B	0	A	0
Value	16^8	16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	16^{-1}	16^{-2}
Results = Value * Digit	0	0	0	0	0	4096	2560	176	0	**0**	***0***
	0	0	0	0	0	4096	2560	176	0	***0.625***	**0**

Conversion: Base *A* -> Base *B*

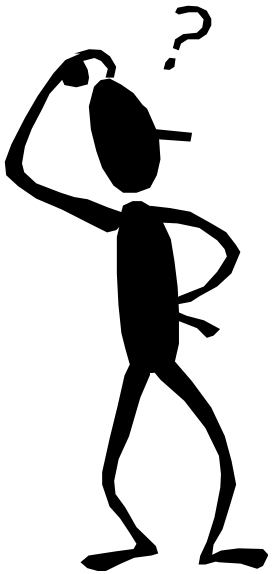
- Radix divide method
  - **Converts** the number in base ***A*** to the equivalent base ***B*** number
  - $(N_I)_A = b_{n-1}B^{n-1} + \ldots + b_1B^1 + b_0B^0$
    - b_i : digits of $(N_I)_B$ in base **A**
  - $(N_I)_A / (B)_A = (b_{n-1}B^{n-1} + \ldots + b_1B^1 + b_0B^0) / (B)_A$
    - $(b_{n-1}B^{n-2} + \ldots + b_1B^0) + b_0$
    - Quotient $Q1 = (b_{n-1}B^{n-2} + \ldots + b_1B^0)$
    - Reminder $R_0 = b_0$
    - $(b_i)_A$ = remainder, R_i , when Q_i is divided by $(B)_A$

Conversion: Base *A* -> Base *B* (cont'd)

- Conversion procedure
  - **Divide** $(N_I)_A$ by $(B)_A$
    - **Produce** Q_1 and R_0
      - R_0 = Least significant digit, b_0
  - **Compute** b_i for I = 1,…., n-1
    - **Divide** Q_i by $(B)_A$
    - **Produce** Q_{i+1} and R_i
      - R_i = digit b_i
  - **Stop**
    - **$Q_{i+1} = 0$**

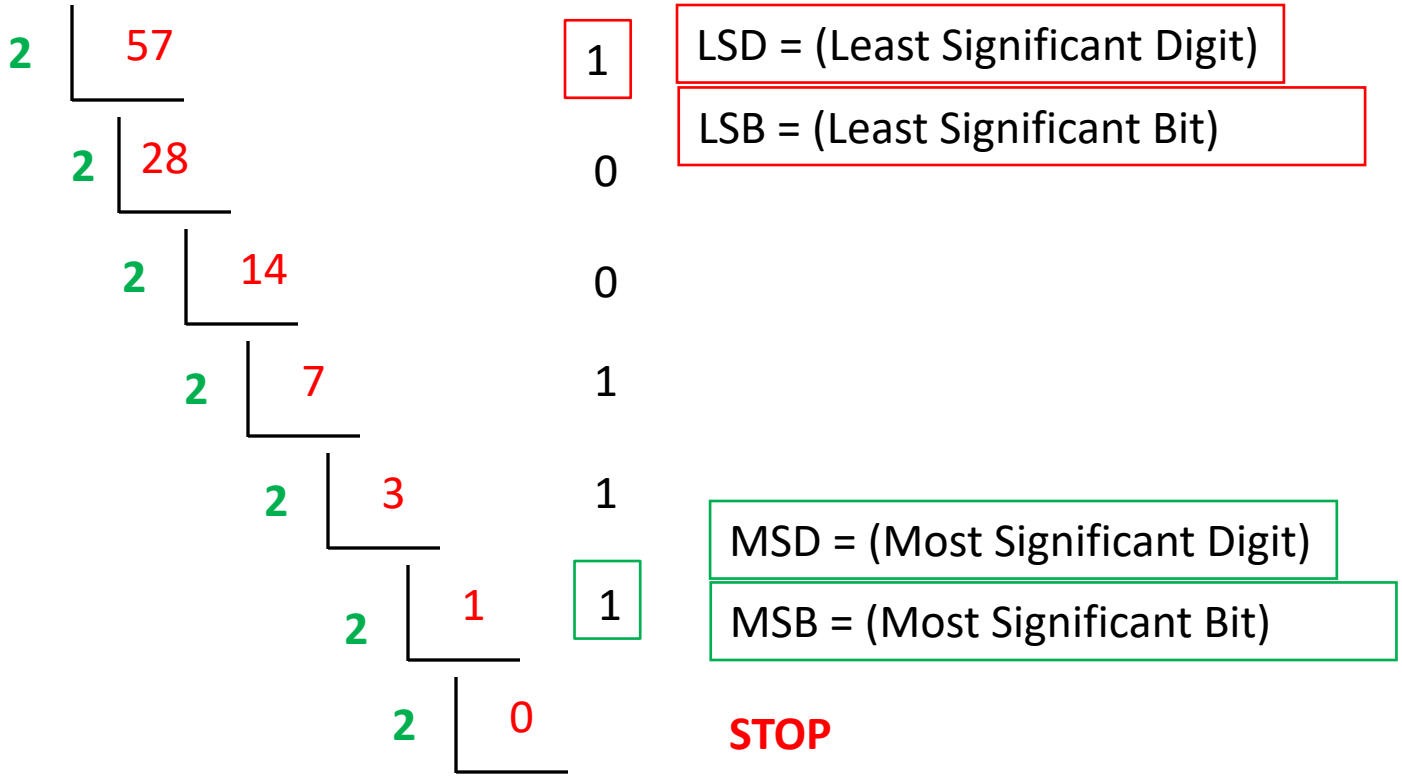
Conversion (A-> B):Sample 5

- $(57)_{10} = (?)_{2}$



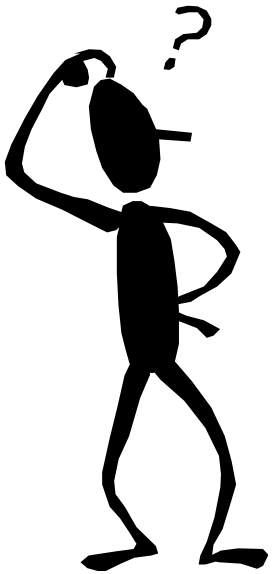
Number-Base Conversion: Sample 1 (cont'd)

- $(57)_{10} = (\boxed{1}\ 1100\ \boxed{1})_2$



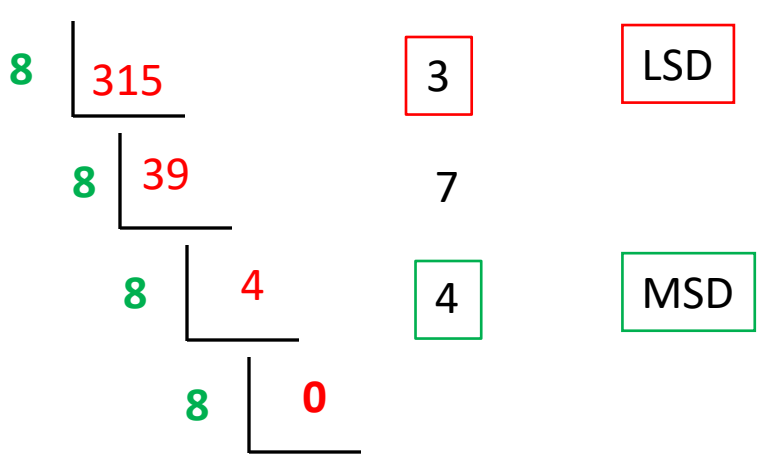
Conversion (A-> B):Sample 6

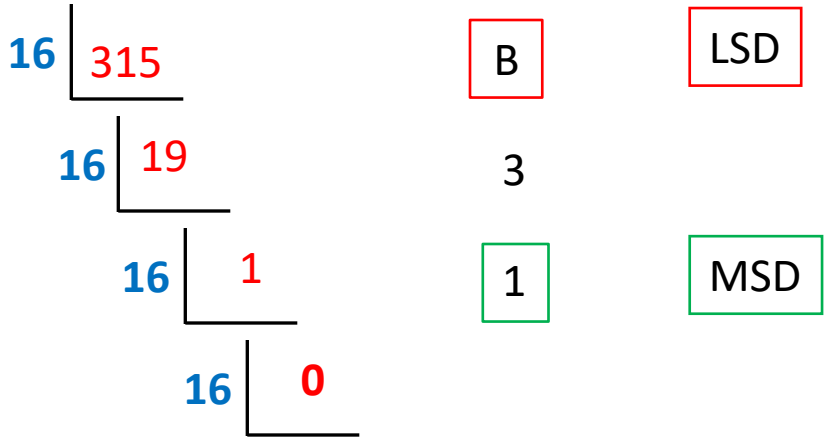
- $(315)_{10} = (?)_{8}$
- $(315)_{10} = (?)_{16}$



Conversion (A-> B):Sample 6(cont'd)

- $(315)_{10} = (473)_8$
- $(315)_{10} = (13B)_{16}$
-



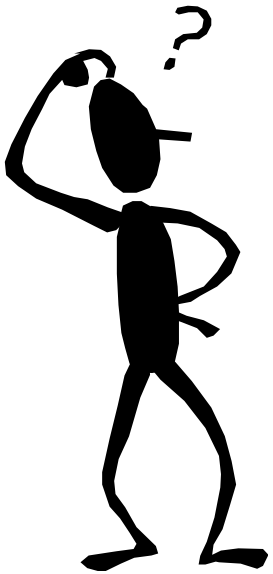


Fraction Numbers

- Radix multiply method
  - Converts the integer in base A to the equivalent base B integer
    - $(N_F)_A = b_{-1}B^{-1} + b_{-2}B^{-2} + \ldots + b_{-m+1}B^{-m+1} + b_{-m}B^{-m}$
      - b_i : digits of $(N_F)_B$ in base **A**
    - $B * N_F = B * (b_{-1}B^{-1} + b_{-2}B^{-2} + \ldots + b_{-m+1}B^{-(m-1)} + b_{-m}B^{-m})$
      - $b_{-1} + (b_{-2}B^{-1} + \ldots + b_{-m+1}B^{-(m-2)} + b_{-m}B^{-(m-1)})$
      - Fraction $F_{-2} = (b_{-2}B^{-1} + \ldots + b_{-m+1}B^{-(m-2)} + b_{-m}B^{-(m-1)})$
      - Integer $I_{-1} = b_{-1}$
    - $(b_i)_A$ = integer part, I_{-i} , of the product of $F_{-(i+1)} \times (B)_A$
  - **Conversion procedure**

Fraction Numbers: Sample 7

- $(0.479)_{10} = (?)_{8}$
- $(0.479)_{10} = (?)_{2}$



Fraction Numbers: Sample 7

- $(0.479)_{10} = (0.\ 3\ 6\ 5\ 1)_8$

$3.832 \leftarrow 0.479 \times 8$ MSD

$6.656 \leftarrow 0.832 \times 8$

$5.248 \leftarrow 0.656 \times 8$

$1.984 \leftarrow 0.248 \times 8$ LSD

...

- $(0.479)_{10} = (0.\ 0\ 1\ 1\ 1)_2$

$0.9580 \leftarrow 0.479 \times 2$ MSD

$1.9160 \leftarrow 0.9580 \times 2$

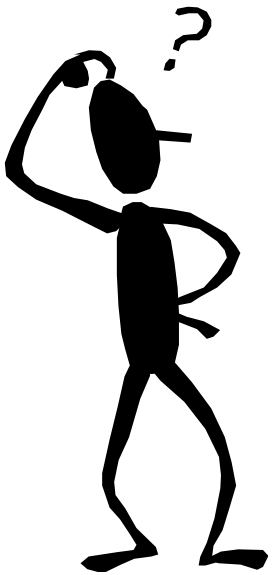
$1.8320 \leftarrow 0.9160 \times 2$

$1.6640 \leftarrow 0.8320 \times 2$ LSD

...

Convert Numbers

- $(254)_{10} = (?)_{2}$
- $(254)_{10} = (?)_{8}$
- $(254)_{10} = (?)_{16}$
- $(0.254)_{10} = (?)_{2}$
- $(0.254)_{10} = (?)_{8}$
- $(0.254)_{10} = (?)_{16}$
- $(254.254)_{10} = (?)_{2}$
- $(254.254)_{10} = (?)_{8}$
- $(254.254)_{10} = (?)_{16}$



Example (cont'd)

- $(254)_{10} = (\ 11111110\ )_2$
  - $254 = 128 + 126$
  - $126 = 64 + 62$
  - $62 = 32 + 30$
  - $30 = 16 + 14$
  - $14 = 8 + 6$
  - $6 = 4 + 2$
  - $2 = 2 + 0$
- $(254)_{10} = (?)_8$
  - $(254)_{10} = (\ 11\ \ 111\ \ 110\ )_2$
  - $(254)_{10} = (\ 3\ \ \ 7\ \ \ 6\ )_8$
- $(254)_{10} = (?)_{16}$
  - $(254)_{10} = (1111\ \ 1110\ )_2$
  - $(254)_{10} = (\ \ F\ \ \ \ E)_{16}$

Example (cont'd)

- $(0.254)_{10} = (?)_2$
  - $0.508 \leftarrow 0.254 \times \mathbf{2}$
  - $1.016 \leftarrow 0.508 \times \mathbf{2}$
  - $0.032 \leftarrow 0.016 \times \mathbf{2}$
  - $0.064 \leftarrow 0.032 \times \mathbf{2}$
- $(0.254)_{10} = (0.0100)_2$

- $(0.254)_{10} = (?)_8$
  - $2.032 \leftarrow 0.254 \times \mathbf{8}$
  - $0.256 \leftarrow 0.032 \times \mathbf{8}$
  - $2.048 \leftarrow 0.256 \times \mathbf{8}$
  - $0.384 \leftarrow 0.048 \times \mathbf{8}$
- $(0.254)_{10} = (0.2020)_8$
  - $(0.254)_{10} = (0.010\ 0)_2$
  - $(0.254)_{10} = (0.2\ 0)_8$

- $(0.254)_{10} = (?)_{16}$
  - $4.064 \leftarrow 0.254 \times \mathbf{16}$
  - $1.024 \leftarrow 0.064 \times \mathbf{16}$
  - $0.384 \leftarrow 0.024 \times \mathbf{16}$
  - $6.144 \leftarrow 0.354 \times \mathbf{16}$
- $(0.254)_{10} = (0.4106)_{16}$
  - $(0.254)_{10} = (0.0100)_2$
  - $(0.254)_{10} = (0.4)_{16}$

Example (cont'd)

- $(254.254)_{10} = (?)_2$
  - $(254)_{10} = (11111110)_2$
  - $(0.254)_{10} = (\mathbf{0.0100})_2$
  - $= (11111110.\mathbf{0100})_2$
- $(254.254)_{10} = (?)_8$
  - $(254)_{10} = (376)_8$
  - $(0.254)_{10} = (\mathbf{0.0020})_8$
  - $= (376.\mathbf{0020})_2$
- $(254.254)_{10} = (?)_{16}$
  - $(254)_{10} = (FE)_8$
  - $(0.254)_{10} = (\mathbf{0.4106})_8$
  - $= (FE.\mathbf{4106})_2$

Thank You