Digital Logic Design

Hajar Falahati

Department of Computer Engineering
IRAN University of Science and Technology

hfalahati@iust.ac.ir

Simplification

- Goal
  - Minimize the cost of realizing a switching function
- Cost measures and other considerations
  - Number of gates
  - Number of levels
  - Gate fan in and/or fan out
  - Interconnection complexity
  - Preventing hazards
- Two-level realizations
  - Minimize the number of gates (terms in switching function)
  - Minimize the fan in (literals in switching function)

Outline

- Quine-McCluskey

Quine-McCluskey

Quine-McCluskey (Q-M)

- A tabular approach to boolean function minimization
- Advantages over K-maps
  - Can be computerized
  - Can handle functions of more than six variables

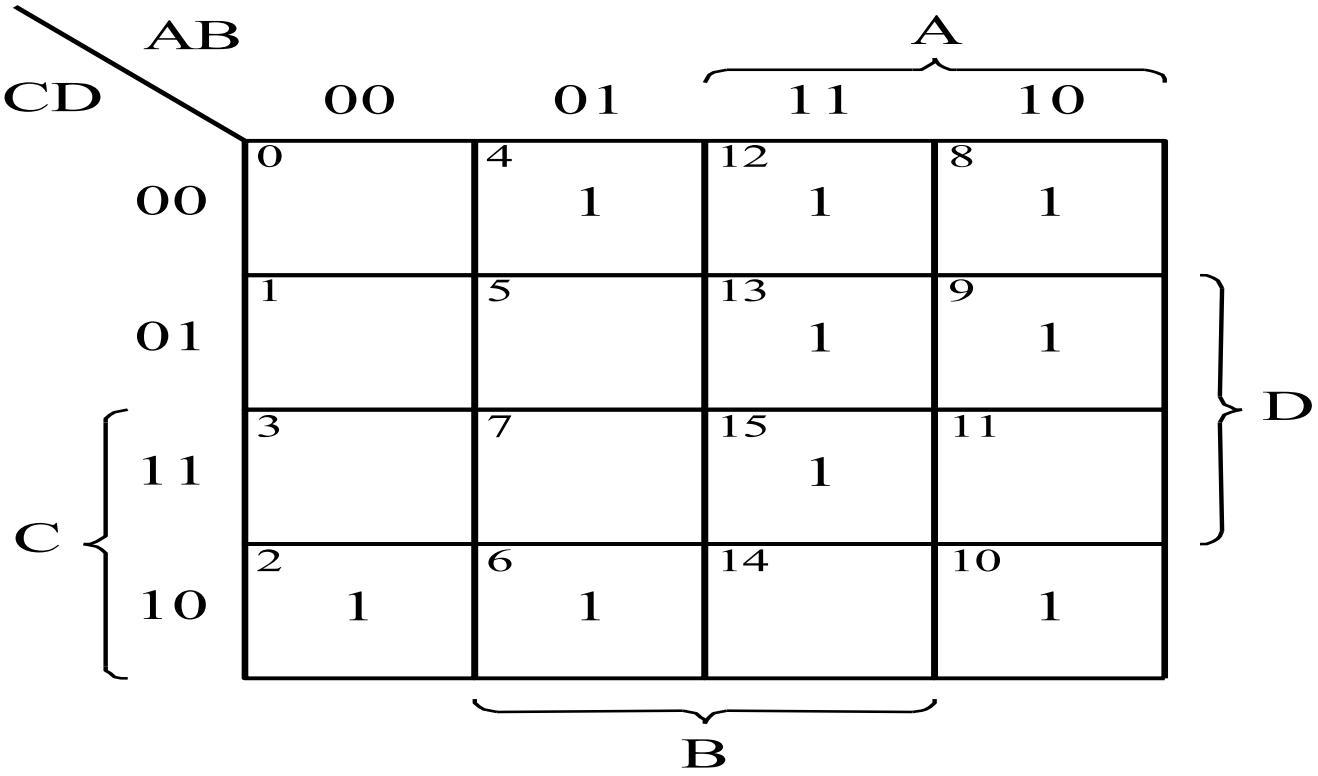
Quine-McCluskey: Overview

- Input: the minterms of a function
- Find all prime implicants (steps 1 and 2)
  - Partition minterms into groups according to the number of 1's
  - Exhaustively search for prime implicants
- Find a minimum prime implicant cover (steps 3 and 4)
  - Construct a prime implicant chart
  - Select the minimum number of prime implicants

Quine-McCluskey: Sample

- Find the MSOP of the function

$f(A,B,C,D) = \sum m(2,4,6,8,9,10,12,13,15)$



Step1: List of Minterms

$f(A,B,C,D) = \sum m(2,4,6,8,9,10,12,13,15)$

Minterms	ABCD
2	0010
4	0100
6	0110
8	1000
9	1001
10	1010
12	1100
13	1101
15	1111

Step1: Partitioning: One 1's

- Partition minterms into groups with 1 1's

Minterms	ABCD
2	0010
4	0100
6	0110
8	1000
9	1001
10	1010
12	1100
13	1101
15	1111

Minterms	ABCD	comment
2	0010	Group 1 (a single 1)
4	0100	
8	1000	

Step1: Partitioning: Two 1's

- Partition minterms into groups with 2 1's

Minterms	ABCD
2	0010
4	0100
6	0110
8	1000
9	1001
10	1010
12	1100
13	1101
15	1111

Minterms	ABCD	comment
2	0010	Group 1 (a single 1)
4	0100	
8	1000	

Step1: Partitioning: Three 1's

- Partition minterms into groups with 3 1's

Minterms	ABCD
2	0010
4	0100
6	0110
8	1000
9	1001
10	1010
12	1100
13	1101
15	1111

Minterms	ABCD	comment
2	0010	Group 1 (a single 1)
4	0100	
8	1000	
6	0110	Group 2 (two 1's)
9	1001	
10	1010	
12	1100	

Step1: Partitioning: Four 1's

- Partition minterms into groups with 4 1's

Minterms	ABCD
2	0010
4	0100
6	0110
8	1000
9	1001
10	1010
12	1100
13	1101
15	1111

Minterms	ABCD	comment
2	0010	Group 1 (a single 1)
4	0100	
8	1000	
6	0110	Group 2 (two 1's)
9	1001	
10	1010	
12	1100	
13	1101	Group 3 (three 1's)

Step1: Partitioning: DONE

- DONE ☺

Minterms	ABCD
2	0010
4	0100
6	0110
8	1000
9	1001
10	1010
12	1100
13	1101
15	1111

Minterms	ABCD	comment
2	0010	Group 1 (a single 1)
4	0100	
8	1000	
6	0110	Group 2 (two 1's)
9	1001	
10	1010	
12	1100	
13	1101	Group 3 (three 1's)
15	1111	Group 4 (four 1's)

Step2: Generate Prime Implicants

- Consider one group with its neighbor group

Prime Implicants

	Minterms	ABCD		Minterms	ABCD
	2	0010			
(1 1's)	4	0100			
	8	1000			
	6	0110			
	9	1001			
(2 1's)	10	1010			
	12	1100			
(3 1's)	13	1101			
(4 1's)	15	1111			

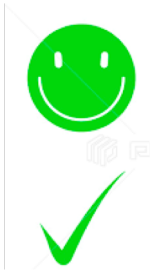
Step2:
Group 1 with Group 2

- m_2 of group 1
  - m_6 of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
	6	0110	
(2 1's)	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10



Step2:
Group 1 with Group 2

- m_2 of group 1
  - m_9 of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
	6	0110	
(2 1's)	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10

Step2:
Group 1 with Group 2

- m_2 of group 1
  - m_{10} of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
	6	0110	
	9	1001	
(2 1's)	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10
2,10	-010

Step2:
Group 1 with Group 2

- m_2 of group 1
  - m_{12} of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10
2,10	-010

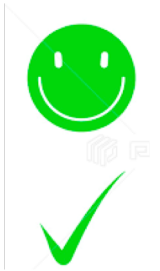
Step2:
Group 1 with Group 2

- m_4 of group 1
  - m_6 of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
	6	0110	
(2 1's)	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10
2,10	-010
4,6	01-0



Step2:
Group 1 with Group 2

- m_4 of group 1
  - m_9 of group 2

	Minterms	ABCD	
	2	0010	
(1 1's)	4	0100	
	8	1000	
	6	0110	
(2 1's)	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10
2,10	-010
4,6	01-0

Step2:
Group 1 with Group 2

- m_4 of group 1
  - m_{10} of group 2

	Minterms	ABCD	
	2	0010	
(1 1's)	4	0100	
	8	1000	
	6	0110	
	9	1001	
(2 1's)	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10
2,10	-010
4,6	01-0

Step2:
Group 1 with Group 2

- m_4 of group 1
  - m_{12} of group 2

	Minterms	ABCD	
	2	0010	
(1 1's)	4	0100	
	8	1000	
	6	0110	
	9	1001	
(2 1's)	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10
2,10	-010
4,6	01-0
4,12	-100

Step2:
Group 1 with Group 2

- m_8 of group 1
  - m_6 of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
	6	0110	
(2 1's)	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10
2,10	-010
4,6	01-0
4,12	-100

Step2:
Group 1 with Group 2

- m_8 of group 1
  - m_9 of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
	6	0110	
(2 1's)	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10
2,10	-010
4,6	01-0
4,12	-100
8,9	100-

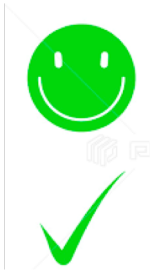
Step2:
Group 1 with Group 2

- m_8 of group 1
  - m_{10} of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
	6	0110	
(2 1's)	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10
2,10	-010
4,6	01-0
4,12	-100
8,9	100-
8,10	10-0



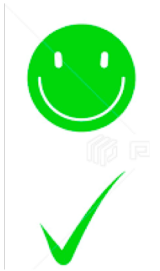
Step2:
Group 1 with Group 2

- m_8 of group 1
  - m_{12} of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
	6	0110	
(2 1's)	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10
2,10	-010
4,6	01-0
4,12	-100
8,9	100-
8,10	10-0
8,12	1-00



Step2:
Group 2 with Group 3

- m_6 of group 2
  - m_{13} of group 3

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10
2,10	-010
4,6	01-0
4,12	-100
8,9	100-
8,10	10-0
8,12	1-00

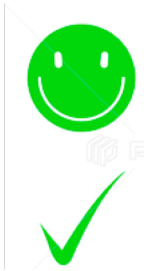
Step2:
Group 2 with Group 3

- m_9 of group 2
  - m_{13} of group 3

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
	6	0110	
(2 1's)	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10
2,10	-010
4,6	01-0
4,12	-100
8,9	100-
8,10	10-0
8,12	1-00
9,13	1-01



Step2:
Group 2 with Group 3

- m_{10} of group 2
  - m_{13} of group 3

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10
2,10	-010
4,6	01-0
4,12	-100
8,9	100-
8,10	10-0
8,12	1-00
9,13	1-01

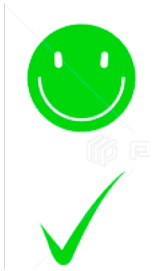
Step2:
Group 2 with Group 3

- m_{12} of group 2
  - m_{13} of group 3

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10
2,10	-010
4,6	01-0
4,12	-100
8,9	100-
8,10	10-0
8,12	1-00
9,13	1-01
12,13	110-



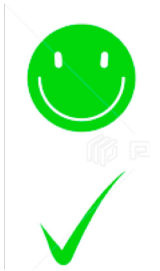
Step2:
Group 3 with Group 4

- m_{13} of group 3
  - m_{15} of group 4

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD
2,6	0-10
2,10	-010
4,6	01-0
4,12	-100
8,9	100-
8,10	10-0
8,12	1-00
9,13	1-01
12,13	110-
13,15	11-1



Step2: Partitioning in New Groups

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	(1 1's)
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	(2 1's)
12,13	110-	
13,15	11-1	(3 1's)

Step2: Partitioning in New Groups

- (2,6) of group 1
  - (9,13) of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	(1 1's)
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	
12,13	110-	(2 1's)
13,15	11-1	(3 1's)

Minterms	ABCD

Step2: Partitioning in New Groups

- (2,6) of group 1
  - (12,13) of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	(1 1's)
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	
12,13	110-	(2 1's)
13,15	11-1	(3 1's)

Minterms	ABCD

Step2: Partitioning in New Groups

- (2,10) of group 1
  - (9,13) of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	(1 1's)
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	
12,13	110-	(2 1's)
13,15	11-1	(3 1's)

Minterms	ABCD

Step2: Partitioning in New Groups

- (2,10) of group 1
  - (12,13) of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	(1 1's)
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	
12,13	110-	(2 1's)
13,15	11-1	(3 1's)

Minterms	ABCD

Step2: Partitioning in New Groups

- (4,6) of group 1
  - (9,13) of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	(1 1's)
4,12	-100	
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	(2 1's)
12,13	110-	
13,15	11-1	(3 1's)

Minterms	ABCD

Step2: Partitioning in New Groups

- (4,6) of group 1
  - (12,13) of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	(1 1's)
4,12	-100	
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	
12,13	110-	(2 1's)
13,15	11-1	(3 1's)

Minterms	ABCD

Step2: Partitioning in New Groups

- (4,12) of group 1
  - (9,13) of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	(1 1's)
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	
12,13	110-	(2 1's)
13,15	11-1	(3 1's)

Minterms	ABCD

Step2: Partitioning in New Groups

- (4,12) of group 1
  - (12,13) of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	(1 1's)
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	
12,13	110-	(2 1's)
13,15	11-1	(3 1's)

Minterms	ABCD

Step2: Partitioning in New Groups

- (8,9) of group 1
  - (9,13) of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	(1 1's)
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	
12,13	110-	(2 1's)
13,15	11-1	(3 1's)

Minterms	ABCD

Step2: Partitioning in New Groups

- (8,9) of group 1
  - (12,13) of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	(1 1's)
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	
12,13	110-	(2 1's)
13,15	11-1	(3 1's)

Minterms	ABCD
8,9,12,13	1-0-

Step2: Partitioning in New Groups

- (8,10) of group 1
  - (9,13) of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	(1 1's)
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	
12,13	110-	(2 1's)
13,15	11-1	(3 1's)

Minterms	ABCD
8,9,12,13	1-0-

Step2: Partitioning in New Groups

- (8,10) of group 1
  - (12,13) of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	(1 1's)
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	
12,13	110-	(2 1's)
13,15	11-1	(3 1's)

Minterms	ABCD
8,9,12,13	1-0-

Step2: Partitioning in New Groups

- (8,12) of group 1
  - (9,13) of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	(1 1's)
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	
12,13	110-	(2 1's)
13,15	11-1	(3 1's)

Minterms	ABCD
8,9,12,13	1-0-
8,12,9,13	1-0-

Duplicated!

Step2: Partitioning in New Groups

- (8,12) of group 1
  - (12,13) of group 2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	(1 1's)
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	
12,13	110-	(2 1's)
13,15	11-1	(3 1's)

Minterms	ABCD
8,9,12,13	1-0-

Step2: Partitioning in New Groups

- (9,13) of group 2
  - (13,15) of group 3

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	(1 1's)
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	
12,13	110-	(2 1's)
13,15	11-1	(3 1's)

Minterms	ABCD
8,9,12,13	1-0-

Step2: Partitioning in New Groups

- (12,13) of group 2
  - (13,15) of group 3

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	(1 1's)
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	
12,13	110-	(2 1's)
13,15	11-1	(3 1's)

Minterms	ABCD
8,9,12,13	1-0-

Step 2: Prime Implicant Cover: PI_1

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	
8,9	100-	
8,10	10-0	
8,12	1-00	
9,13	1-01	
12,13	110-	
13,15	11-1	

(1 1's)

(2 1's)

(3 1's)

Prime Implicants

Minterms	ABCD	
8,9,12,13	1-0-	PI_1

Step 2: Prime Implicant Cover: PI_1 (cont'd)

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	
(2 1's)	6	0110	
	9	1001	
	10	1010	
	12	1100	
(3 1's)	13	1101	
(4 1's)	15	1111	

Prime Implicants

	Minterms	ABCD	
	2,6	0-10	
	2,10	-010	
	4,6	01-0	
	4,12	-100	
	8,9	100-	√
	8,10	10-0	
	8,12	1-00	
	9,13	1-01	
	12,13	110-	√
	13,15	11-1	

Prime Implicants

	Minterms	ABCD	
	8,9,12,13	1-0-	PI_1
(1 1's)			
(2 1's)			
(3 1's)			

Step 2: Prime Implicant Cover: PI_1 (cont'd)

	Minterms	ABCD	
	2	0010	
(1 1's)	4	0100	
	8	1000	√
	6	0110	
(2 1's)	9	1001	√
	10	1010	
	12	1100	√
(3 1's)	13	1101	√
(4 1's)	15	1111	

Prime Implicants

Minterms	ABCD	
2,6	0-10	
2,10	-010	
4,6	01-0	
4,12	-100	
8,9	100-	√
8,10	10-0	
8,12	1-00	√
9,13	1-01	√
12,13	110-	√
13,15	11-1	

(1 1's)

(2 1's)

(3 1's)

Prime Implicants

Minterms	ABCD	
8,9,12,13	1-0-	PI_1

Step 2: Prime Implicant Cover: PI_2

	Minterms	ABCD	
(1 1's)	2	0010	
	4	0100	
	8	1000	√
(2 1's)	6	0110	
	9	1001	√
	10	1010	
	12	1100	√
(3 1's)	13	1101	√
(4 1's)	15	1111	

Prime Implicants

	Minterms	ABCD	
	2,6	0-10	PI_2
	2,10	-010	
	4,6	01-0	
(1 1's)	4,12	-100	
	8,9	100-	√
	8,10	10-0	
	8,12	1-00	√
	9,13	1-01	√
(2 1's)	12,13	110-	√
(3 1's)	13,15	11-1	

Prime Implicants

Minterms	ABCD	
8,9,12,13	1-0-	PI_1

Step 2: Prime Implicant Cover: PI_2 (cont'd)

	Minterms	ABCD	
(1 1's)	2	0010	√
	4	0100	
	8	1000	√
(2 1's)	6	0110	√
	9	1001	√
	10	1010	
	12	1100	√
(3 1's)	13	1101	√
(4 1's)	15	1111	

Prime Implicants

	Minterms	ABCD	
	2,6	0-10	PI_2
	2,10	-010	
	4,6	01-0	
(1 1's)	4,12	-100	
	8,9	100-	√
	8,10	10-0	
	8,12	1-00	√
	9,13	1-01	√
(2 1's)	12,13	110-	√
(3 1's)	13,15	11-1	

Prime Implicants

Minterms	ABCD	
8,9,12,13	1-0-	PI_1

Step 2: Prime Implicant Cover: PI_3

	Minterms	ABCD	
(1 1's)	2	0010	√
	4	0100	
	8	1000	√
(2 1's)	6	0110	√
	9	1001	√
	10	1010	
	12	1100	√
(3 1's)	13	1101	√
(4 1's)	15	1111	

Prime Implicants

	Minterms	ABCD	
	2,6	0-10	PI_2
	2,10	-010	PI_3
	4,6	01-0	
(1 1's)	4,12	-100	
	8,9	100-	√
	8,10	10-0	
	8,12	1-00	√
	9,13	1-01	√
(2 1's)	12,13	110-	√
(3 1's)	13,15	11-1	

Prime Implicants

Minterms	ABCD	
8,9,12,13	1-0-	PI_1

Step 2: Prime Implicant Cover: PI_3 (cont'd)

	Minterms	ABCD	
	2	0010	√
(1 1's)	4	0100	
	8	1000	√
	6	0110	√
	9	1001	√
(2 1's)	10	1010	√
	12	1100	√
(3 1's)	13	1101	√
(4 1's)	15	1111	

Prime Implicants

	Minterms	ABCD	
	2,6	0-10	PI_2
	2,10	-010	PI_3
	4,6	01-0	
(1 1's)	4,12	-100	
	8,9	100-	√
	8,10	10-0	
	8,12	1-00	√
	9,13	1-01	√
(2 1's)	12,13	110-	√
(3 1's)	13,15	11-1	

Prime Implicants

Minterms	ABCD	
8,9,12,13	1-0-	PI_1

Step 2: Prime Implicant Cover: PI_4

	Minterms	ABCD	
(1 1's)	2	0010	√
	4	0100	
	8	1000	√
(2 1's)	6	0110	√
	9	1001	√
	10	1010	√
	12	1100	√
(3 1's)	13	1101	√
(4 1's)	15	1111	

Prime Implicants

	Minterms	ABCD	
	2,6	0-10	PI_2
	2,10	-010	PI_3
	4,6	01-0	PI_4
(1 1's)	4,12	-100	
	8,9	100-	√
	8,10	10-0	
	8,12	1-00	√
(2 1's)	9,13	1-01	√
	12,13	110-	√
(3 1's)	13,15	11-1	

Prime Implicants

Minterms	ABCD	
8,9,12,13	1-0-	PI_1

Step 2: Prime Implicant Cover: PI_4 (cont'd)

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	Minterms	ABCD	
(1 1's)	2	0010	√
	4	0100	√
	8	1000	√
(2 1's)	6	0110	√
	9	1001	√
	10	1010	√
	12	1100	√
(3 1's)	13	1101	√
(4 1's)	15	1111	

Prime Implicants

	Minterms	ABCD	
	2,6	0-10	PI_2
	2,10	-010	PI_3
	4,6	01-0	PI_4
(1 1's)	4,12	-100	
	8,9	100-	√
	8,10	10-0	
	8,12	1-00	√
	9,13	1-01	√
(2 1's)	12,13	110-	√
(3 1's)	13,15	11-1	

Prime Implicants

Minterms	ABCD	
8,9,12,13	1-0-	PI_1

Step 2: Prime Implicant Cover: PI_5

	Minterms	ABCD	
(1 1's)	2	0010	√
	4	0100	√
	8	1000	√
(2 1's)	6	0110	√
	9	1001	√
	10	1010	√
	12	1100	√
(3 1's)	13	1101	√
(4 1's)	15	1111	

Prime Implicants

	Minterms	ABCD	
	2,6	0-10	PI_2
	2,10	-010	PI_3
	4,6	01-0	PI_4
(1 1's)	4,12	-100	PI_5
	8,9	100-	√
	8,10	10-0	
	8,12	1-00	√
	9,13	1-01	√
(2 1's)	12,13	110-	√
(3 1's)	13,15	11-1	

Prime Implicants

Minterms	ABCD	
8,9,12,13	1-0-	PI_1

Step 2: Prime Implicant Cover: PI_6

	Minterms	ABCD	
(1 1's)	2	0010	√
	4	0100	√
	8	1000	√
(2 1's)	6	0110	√
	9	1001	√
	10	1010	√
	12	1100	√
(3 1's)	13	1101	√
(4 1's)	15	1111	

Prime Implicants

	Minterms	ABCD	
	2,6	0-10	PI_2
	2,10	-010	PI_3
	4,6	01-0	PI_4
(1 1's)	4,12	-100	PI_5
	8,9	100-	√
	8,10	10-0	PI_6
	8,12	1-00	√
	9,13	1-01	√
(2 1's)	12,13	110-	√
(3 1's)	13,15	11-1	

Prime Implicants

Minterms	ABCD	
8,9,12,13	1-0-	PI_1

Step 2: Prime Implicant Cover: PI_7

	Minterms	ABCD	
(1 1's)	2	0010	√
	4	0100	√
	8	1000	√
(2 1's)	6	0110	√
	9	1001	√
	10	1010	√
	12	1100	√
(3 1's)	13	1101	√
(4 1's)	15	1111	

Prime Implicants

	Minterms	ABCD	
	2,6	0-10	PI_2
	2,10	-010	PI_3
	4,6	01-0	PI_4
(1 1's)	4,12	-100	PI_5
	8,9	100-	√
	8,10	10-0	PI_6
	8,12	1-00	√
	9,13	1-01	√
(2 1's)	12,13	110-	√
(3 1's)	13,15	11-1	PI_7

Prime Implicants

Minterms	ABCD	
8,9,12,13	1-0-	PI_1

Step 2: Prime Implicant Cover: PI_7 (cont'd)

	Minterms	ABCD	
(1 1's)	2	0010	√
	4	0100	√
	8	1000	√
(2 1's)	6	0110	√
	9	1001	√
	10	1010	√
	12	1100	√
(3 1's)	13	1101	√
(4 1's)	15	1111	√

Prime Implicants

Minterms	ABCD		
2,6	0-10	PI_2	
2,10	-010	PI_3	
4,6	01-0	PI_4	
4,12	-100	PI_5	(1 1's)
8,9	100-	√	
8,10	10-0	PI_6	
8,12	1-00	√	
9,13	1-01	√	
12,13	110-	√	(2 1's)
13,15	11-1	PI_7	(3 1's)

Prime Implicants

Minterms	ABCD	
8,9,12,13	1-0-	PI_1

Step3

- Goal: Find minimum Coverage

- PI_1 : 8,9,12,13
- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10
- PI_7 : 13,15

Step3

- Construct a prime implicant chart

- PI_1 : 8,9,12,13
- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10
- PI_7 : 13,15

	2	4	6	8	9	10	12	13	15
PI_1									
PI_2									
PI_3									
PI_4									
PI_5									
PI_6									
PI_7									

Step3

- Construct a prime implicant chart

- PI_1 : 8,9,12,13
- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10
- PI_7 : 13,15

	2	4	6	8	9	10	12	13	15
PI_1				×	×		×	×	
PI_2									
PI_3									
PI_4									
PI_5									
PI_6									
PI_7									

Step3

- Construct a prime implicant chart

- PI_1 : 8,9,12,13
- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10
- PI_7 : 13,15

	2	4	6	8	9	10	12	13	15
PI_1				×	×		×	×	
PI_2	×		×						
PI_3									
PI_4									
PI_5									
PI_6									
PI_7									

Step3

- Construct a prime implicant chart

- PI_1 : 8,9,12,13
- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10
- PI_7 : 13,15

	2	4	6	8	9	10	12	13	15
PI_1				×	×		×	×	
PI_2	×		×						
PI_3	×					×			
PI_4									
PI_5									
PI_6									
PI_7									

Step3

- Construct a prime implicant chart

- PI_1 : 8,9,12,13
- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10
- PI_7 : 13,15

	2	4	6	8	9	10	12	13	15
PI_1				×	×		×	×	
PI_2	×		×						
PI_3	×					×			
PI_4		×	×						
PI_5									
PI_6									
PI_7									

Step3

- Construct a prime implicant chart

- PI_1 : 8,9,12,13
- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10
- PI_7 : 13,15

	2	4	6	8	9	10	12	13	15
PI_1				×	×		×	×	
PI_2	×		×						
PI_3	×					×			
PI_4		×	×						
PI_5		×					×		
PI_6									
PI_7									

Step3

- Construct a prime implicant chart

- PI_1 : 8,9,12,13
- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10
- PI_7 : 13,15

	2	4	6	8	9	10	12	13	15
PI_1				×	×		×	×	
PI_2	×		×						
PI_3	×					×			
PI_4		×	×						
PI_5		×					×		
PI_6				×		×			
PI_7									

Step3

- Construct a prime implicant chart

- PI_1 : 8,9,12,13
- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10
- PI_7 : 13,15

	2	4	6	8	9	10	12	13	15
PI_1				×	×		×	×	
PI_2	×		×						
PI_3	×					×			
PI_4		×	×						
PI_5		×					×		
PI_6				×		×			
PI_7								×	×

Step3

- Find the essential prime implicant

- PI_1 : 8,9,12,13
- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10
- PI_7 : 13,15

	2	4	6	8	9	10	12	13	15
PI_1				×	×		×	×	
PI_2	×		×						
PI_3	×					×			
PI_4		×	×						
PI_5		×					×		
PI_6				×		×			
PI_7								×	×

Step3

- Find the essential prime implicant

- PI_1 : 8,9,12,13
- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10
- PI_7 : 13,15

	2	4	6	8	9	10	12	13	15
PI_1				×	(×)		×	×	
PI_2	×		×						
PI_3	×					×			
PI_4		×	×						
PI_5		×					×		
PI_6				×		×			
PI_7								×	(×)

Step3

- Find the essential prime implicant

- PI_1 : 8,9,12,13
- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10
- PI_7 : 13,15

		2	4	6	8	9	10	12	13	15
**	PI_1				×	×		×	×	
	PI_2	×		×						
	PI_3	×					×			
	PI_4		×	×						
	PI_5		×					×		
	PI_6				×		×			
	PI_7								×	×

Step3

- Find the essential prime implicant

- PI_1 : 8,9,12,13
- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10
- PI_7 : 13,15

					√	√		√	√	
	2	4	6	8	9	10	12	13	15
PI_1				×	(×)		×	×	
PI_2	×		×						
PI_3	×					×			
PI_4		×	×						
PI_5		×					×		
PI_6				×		×			
PI_7								×	(×)

Step3

- Find the essential prime implicant

- PI_1 : 8,9,12,13
- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10
- PI_7 : 13,15

					√	√		√	√	
		2	4	6	8	9	10	12	13	15
**	PI_1				×	×		×	×	
	PI_2	×		×						
	PI_3	×					×			
	PI_4		×	×						
	PI_5		×					×		
	PI_6				×		×			
**	PI_7								×	×

Step3

- Find the essential prime implicant

- PI_1 : 8,9,12,13
- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10
- PI_7 : 13,15

					√	√		√	√	√
		2	4	6	8	9	10	12	13	15
**	PI_1				×	×		×	×	
	PI_2	×		×						
	PI_3	×					×			
	PI_4		×	×						
	PI_5		×					×		
	PI_6				×		×			
**	PI_7								×	×

Step4

- Select the minimum number of prime implicants

- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10

	2	4	6	10
PI_2				
PI_3				
PI_4				
PI_5				
PI_6				

Step4

- Select the minimum number of prime implicants

- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10

	2	4	6	10
PI_2	×		×	
PI_3				
PI_4				
PI_5				
PI_6				

Step4

- Select the minimum number of prime implicants

- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10

	2	4	6	10
PI_2	×		×	
PI_3	×			×
PI_4				
PI_5				
PI_6				

Step4

- Select the minimum number of prime implicants

- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10

	2	4	6	10
PI_2	×		×	
PI_3	×			×
PI_4		×	×	
PI_5				
PI_6				

Step4

- Select the minimum number of prime implicants

- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10

	2	4	6	10
PI_2	×		×	
PI_3	×			×
PI_4		×	×	
PI_5		×		
PI_6				

Step4

- Select the minimum number of prime implicants

- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10

	2	4	6	10
PI_2	×		×	
PI_3	×			×
PI_4		×	×	
PI_5		×		
PI_6				×

Step4

- Select the minimum number of prime implicants

- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10

	2	4	6	10
PI_2	×		×	
* PI_3	×			×
PI_4		×	×	
PI_5		×		
PI_6				×

Step4

- Select the minimum number of prime implicants

- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10

	√			√
	2	4	6	10
PI_2	×		×	
* PI_3	×			×
PI_4		×	×	
PI_5		×		
PI_6				×

Step4

- Select the minimum number of prime implicants

- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10

	√			√
	2	4	6	10
PI_2	×		×	
* PI_3	×			×
* PI_4		×	×	
PI_5		×		
PI_6				×

Step4

- Select the minimum number of prime implicants

- PI_2 : 2,6
- PI_3 : 2,10
- PI_4 : 4,6
- PI_5 : 4,12
- PI_6 : 8,10

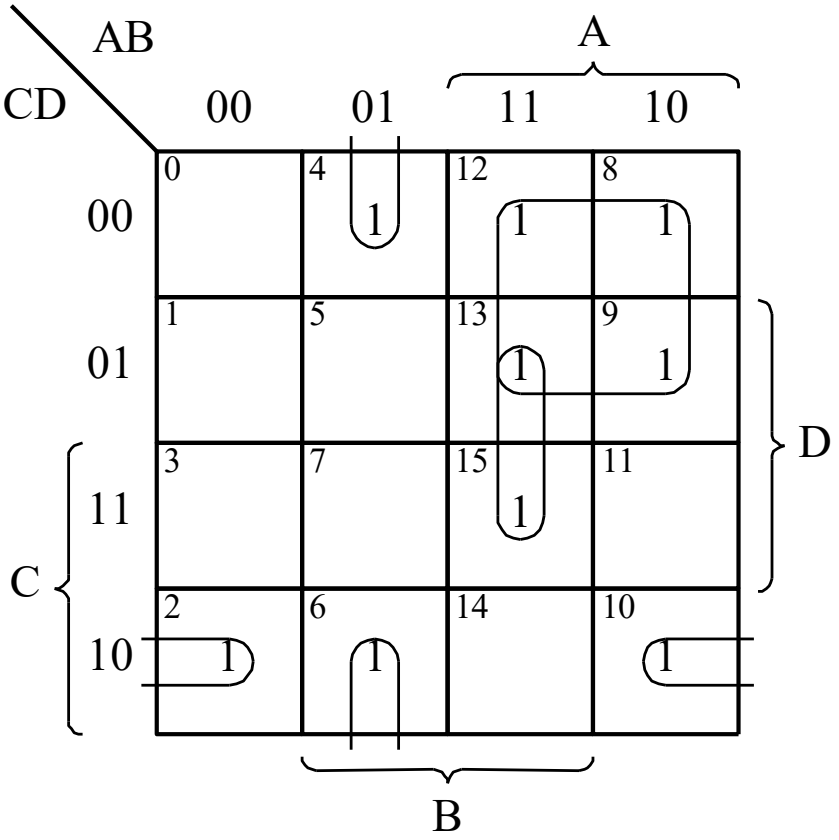
	√	√	√	√
	2	4	6	10
PI_2	×		×	
* PI_3	×			×
* PI_4		×	×	
PI_5		×		
PI_6				×

Step4

- Optimized form

$$f(A,B,C,D) = PI_1 + PI_3 + PI_4 + PI_7$$
$$= 1\text{-}0\text{-} + \text{-}010 + 01\text{-}0 + 11\text{-}1$$
$$= A\overline{C} + \overline{B}C\overline{D} + \overline{A}B\overline{D} + ABD$$

How the Q-M Results Look on a K-map?



Covering Procedure

- Goal
  - Essential implicants
  - Minimum number of prime implicants

					√	√		√	√	√
		2	4	6	8	9	10	12	13	15
**	PI_1				×	×		×	×	
	PI_2	×		×						
	PI_3	×					×			
	PI_4		×	×						
	PI_5		×					×		
	PI_6				×		×			
**	PI_7								×	×

Covering Procedure: Step 1

- Identify any minterms covered by only one PI
- Select these PIs for the cover

	√ 0	√ 1	5	√ 6	√ 7	√ 8	√ 9	10	11	13	√ 14	√ 15
* * PI_1	⊗	×				×	×					
PI_2		×	×				×			×		
PI_3			×		×					×		×
PI_4						×	×	×	×			
PI_5							×		×	×		×
PI_6								×	×		×	×
* * PI_7				⊗	×						×	×

Covering Procedure: Step 2

- Remove rows covered by the PIs identified in step 1
- Remove minterms covered by the removed rows

	√ 0	√ 1	5	√ 6	√ 7	√ 8	√ 9	10	11	13	√ 14	√ 15
* * PI_1	⊗	×				×	×					
PI_2		×	×				×			×		
PI_3			×		×					×		×
PI_4						×	×	×	×			
PI_5							×		×	×		×
PI_6								×	×		×	×
* * PI_7				⊗	×						×	×

	5	10	11	13
PI_2	×			×
PI_3	×			×
PI_4		×	×	
PI_5			×	×
PI_6		×	×	

Covering Procedure: Step 2: Rules

- Rule 1
  - A row that is covered by another row may be eliminated from the chart.
  - Row PI_3 and PI_6 may be eliminated.

- Rule 2
  - A column that covers another column may be eliminated.
  - Columns 11 and 13 can be eliminated

	5	10	11	13
PI_2	×			×
PI_3	×			×
PI_4		×	×	
PI_5			×	×
PI_6		×	×	

Covering Procedure: Step 3

- If a cyclic chart results from step 2, go to step 5.
- Otherwise, apply the reduction procedure of steps 1 and 2.
- Cyclic chart
  - Contains essential PI
  - Cannot reduced by rules 1 and 2

	√ 1	2	√ 3	4	5	6
*PI_1	×		×			
PI_2		×	×			
PI_3		×				×
PI_4				×		×
PI_5				×	×	
PI_6	×				×	

	2	4	5	6
PI_2	×			
PI_3	×			×
PI_4		×		×
PI_5		×	×	
PI_6			×	

	√ 2	√ 4	√ 5	√ 6
*PI_3	×			×
PI_4		×		×
*PI_5		×	×	

Covering Procedure: Step 4-5

- Step 4
  - If a cyclic chart results from step 3, go to step 5.
  - Otherwise return to step 1.
- Step 5
  - Apply the cyclic chart procedure.
  - Repeat step 5 until a void chart or noncyclic chart is produced.
  - In the latter case, return to step 1.

Sample 1

- $f(A,B,C,D) = \sum m(0,1,5,6,7,8,9,10,11,13,14,15)$

Sample1: Partitioning

- $f(A,B,C,D) = \sum m(0,1,5,6,7,8,9,10,11,13,14,15)$

Minterms	ABCD	comment
0	0000	Group 0 (No 1)
1	0001	Group 1 (a single 1)
8	1000	
5	0101	Group 2 (two 1's)
6	0110	
9	1001	
10	1010	
7	0111	Group 3 (three 1's)
11	1011	
13	1101	
14	1110	
15	1111	Group 4 (four 1's)

Sample1: Prime Implicant

Minterms	ABCD	
0	0000	
1	0001	
8	1000	
5	0101	
6	0110	
9	1001	
10	1010	
7	0111	
11	1011	
13	1101	
14	1110	
15	1111	

Minterms	ABCD	
(0,1)	000-	
(1,5)	0-01	
(1,9)	-001	
(9,8)	100-	
(8,10)	10-0	
(5,7)	01-1	
(5,13)	-101	
(6,7)	011-	
(9,11)	10-1	
(10,11)	101-	
(10,14)	1-10	
(7,15)	-111	
(11,15)	1-11	
(13,15)	11-1	
(14,15)	111-	

Minterms	ABCD	
(0,1,8,9)	-00-	PI_1
(1,9,5,13)	--01	PI_2
(5,7,13,15)	-1-1	PI_3
(8,9,10,11)	10--	PI_4
(9,11,13,15)	1--1	PI_5
(10,14,11,15)	1-1-	PI_6
(6,7,14,15)	-11-	PI_7

Sample1: Prime Implicant (cont'd)

Minterms	ABCD	
0	0000	√
1	0001	√
8	1000	√
5	0101	√
6	0110	√
9	1001	√
10	1010	√
7	0111	√
11	1011	√
13	1101	√
14	1110	√
15	1111	√

Minterms	ABCD	
(0,1)	000-	√
(1,5)	0-01	√
(1,9)	-001	√
(9,8)	100-	√
(8,10)	10-0	√
(5,7)	01-1	√
(5,13)	-101	√
(6,7)	011-	√
(9,11)	10-1	√
(10,11)	101-	√
(10,14)	1-10	√
(7,15)	-111	√
(11,15)	1-11	√
(13,15)	11-1	√
(14,15)	111-	√

Minterms	ABCD	
(0,1,8,9)	-00-	PI_1
(1,9,5,13)	--01	PI_2
(5,7,13,15)	-1-1	PI_3
(8,9,10,11)	10--	PI_4
(9,11,13,15)	1--1	PI_5
(10,14,11,15)	1-1-	PI_6
(6,7,14,15)	-11-	PI_7

Sample1: Minimum Coverage

	√ 0	√ 1	5	√ 6	√ 7	√ 8	√ 9	10	11	13	√ 14	√ 15
* * PI_1	⊗	×				×	×					
PI_2		×	×				×			×		
PI_3			×		×					×		×
PI_4						×	×	×	×			
PI_5							×		×	×		×
PI_6								×	×		×	×
* * PI_7				⊗	×						×	×

Sample1: Minimum Coverage (cont'd)

	5	10	11	13
PI_2	×			×
PI_3	×			×
PI_4		×	×	
PI_5			×	×
PI_6		×	×	

	√ 5	√ 10
* PI_2	×	
*PI_4		×

$f(A,B,C,D) = PI_1 + PI_7 + PI_2 + PI_4$

Q-M with Incompletely Specified Functions or Don't Cares

- Use minterms and don't cares when generating prime implicants
- Use only minterms when finding a minimal cover
- Sample

$$f(A,B,C,D,E) = \sum m(2,3,7,10,12,15,27) + d(5,18,19,21,23)$$

Sample 2

$f(A,B,C,D,E) = \sum m(2,3,7,10,12,15,27) + d(5,18,19,21,23)$

Minterm	ABCDE	
2	00010	√
3	00011	√
5	00101	√
10	01010	√
12	01100	PI_7
18	10010	√
7	00111	√
19	10011	√
21	10101	√
15	01111	√
23	10111	√
27	11011	√

Minterm	ABCDE	
2,3	0001-	√
2,10	0-010	PI_4
2,18	-0010	√
3,7	00-11	√
3,19	-0011	√
5, 7	001-1	√
5, 21	-0101	√
18,19	1001-	√
7,15	0-111	PI_5
7, 23	-0111	√
19, 23	10-11	√
19, 27	1-011	PI_6
21,23	101-1	√

Minterm	ABCDE	
2,3,18,19	-001-	PI_1
3,7,19,23	-0-11	PI_2
5,7,21,23	-01-1	PI_3

Sample 2(cont'd)

$f(A,B,C,D,E) = \sum m(2,3,7,10,12,15,27) + d(5,18,19,21,23)$

No Don't care

	√		√	√	√	√	√
	2	3	7	10	12	15	27
PI_1	×	×					
PI_2		×	×				
PI_3			×				
** PI_4	×			⊗			
** PI_5			×			⊗	
** PI_6							⊗
** PI_7					⊗		

	√
	3
** PI_1	×
PI_2	×

Sample 2(cont'd)

$$f(A,B,C,D,E) = PI_1 + PI_4 + PI_5 + PI_6 + PI_7$$

$$f(A,B,C,D,E) = PI_2 + PI_4 + PI_5 + PI_6 + PI_7$$

Sample 3

f(a, b, c, d) =$\prod M(0,8,9,10,14,15) + d(2,6)$

Sample 3 (cont'd)

f(a, b, c, d) = $\prod M(0,8,9,10,14,15) + d(2,6)$

(8,9) 100 - a'+b+c

(14,15) 111- - a'+b'+c'

(0,2,8,10) -0-0 - b+d

f(a, b, c, d) = (a'+b+c) (a'+b'+c') (b+d)

Thank You